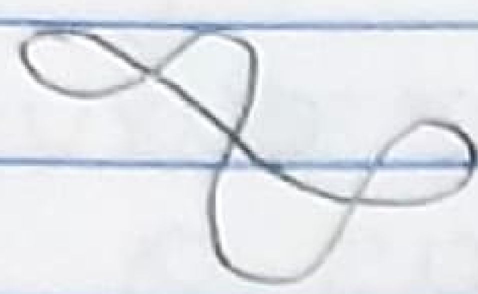
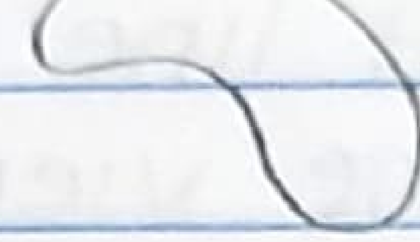
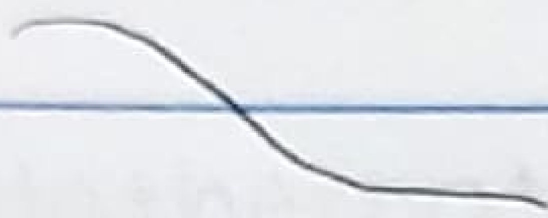
Calculus on a Curve 5/15/23

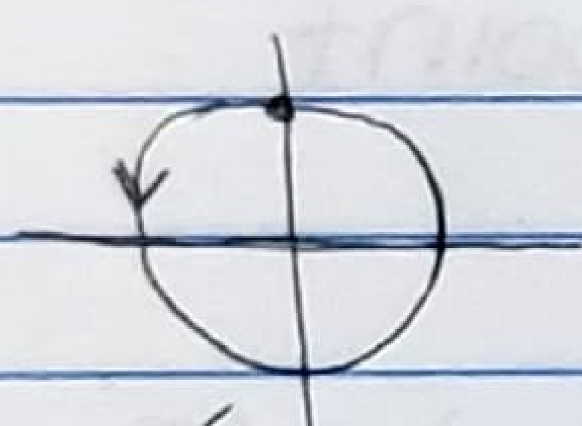
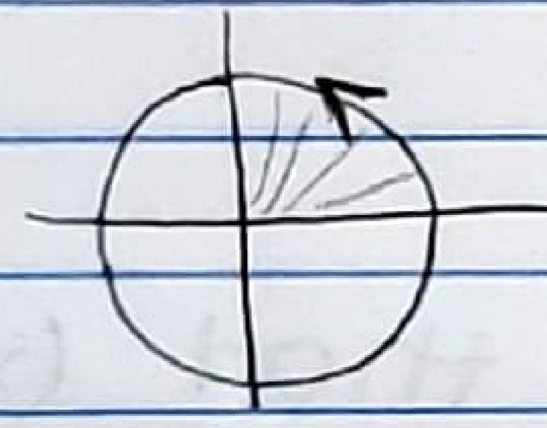
closed curve   closed simple curve (nonintersecting)



not closed

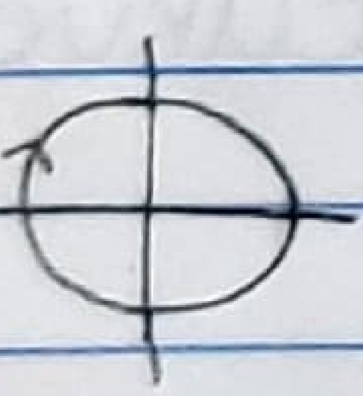
A parametrization of a curve gives the curve an orientation.

*Parametrize a Circle

$\begin{cases} x = \cos t \\ y = \sin t \end{cases}$ $t \in [0, 2\pi]$

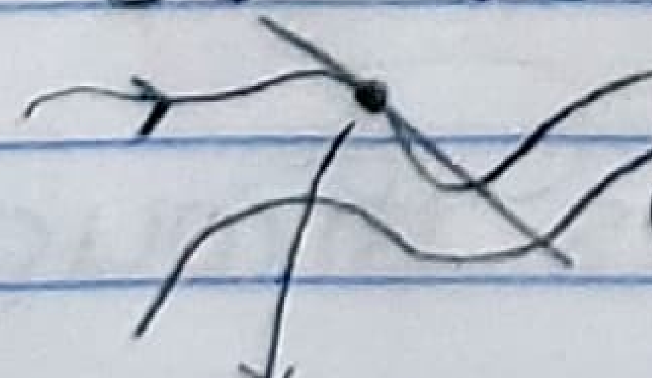
$\begin{cases} x = -\sin t \\ y = \cos t \end{cases}$ $t \in [0, 2\pi]$



$\begin{cases} x = \cos t \\ y = -\sin t \end{cases}$ $t \in [0, 2\pi]$.

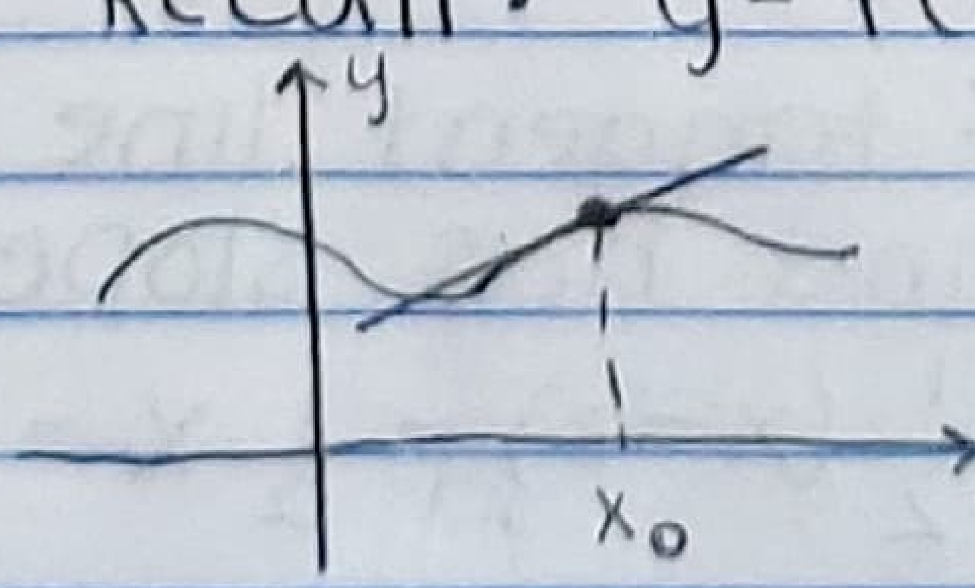
*Tangent Line

$c: \begin{cases} x = x(t) \\ y = y(t) \end{cases}$ parametrization of curve C



coordinate (x, y)
parameter t

Recall: $y = f(x)$



slope $= f'(x_0)$
contain (x_0, y_0)
equation:
$y = y_0 + f'(x_0)(x - x_0)$

Strategy:
To find the tangent line of the curve at point (x_0, y_0), we view y as a function of x when $x \approx x_0$.

$$\text{Slope} = y'(x_0) = \frac{dy}{dx}(x_0)$$

$$y = y(x) = y(x(t))$$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \quad : \text{chain rule}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y'(t_0)}{x'(t_0)}$$

ex. Find the tangent line to the curve $C: x = 2t,\ y = t^2 - t$ at the point $(x, y) = (2, 0)$.

Step 1: Find t such that $(x, y) = (2, 0)$
$t = 1$

Step 2: Find the slope of the curve at $(2, 0)$

$$\frac{dy}{dx}(2) = \frac{dy/dt(1)}{dx/dt(1)} = \frac{1}{2}$$

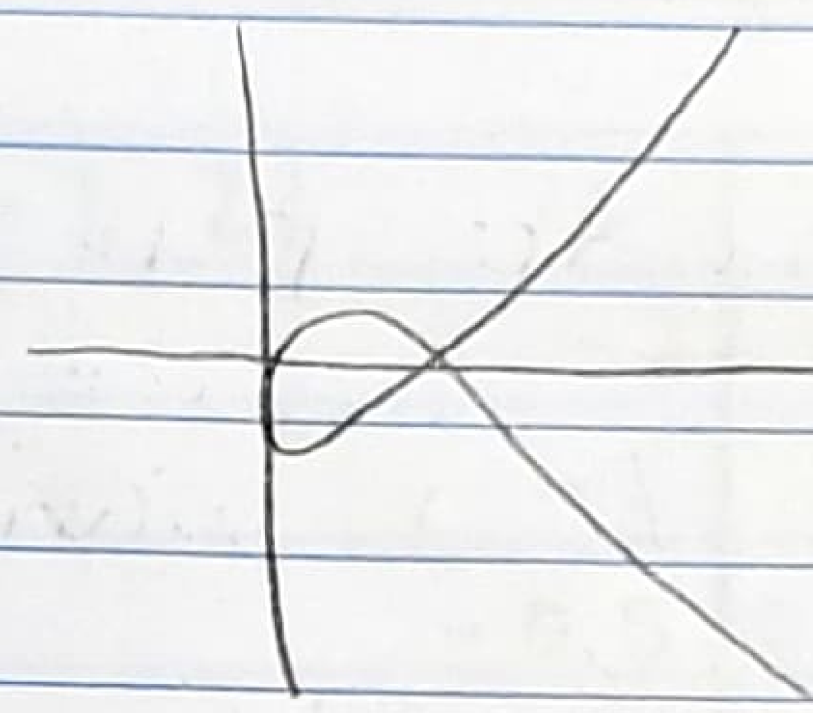
$$\frac{dy}{dt} = 2t - 1 \rightarrow \frac{dy}{dt}(1) = 2(1) - 1 = 1$$

$$\frac{dx}{dt} = 2 \longrightarrow \frac{dx}{dt}(1) = 2$$

Step 3: tangent line passes through $(2, 0)$ and has slope $1/2$

$$y = 0 + \frac{1}{2}(x - 2) = \frac{1}{2}x - 1 \qquad y = \frac{1}{2}x - 1$$

$$\begin{cases} x = t^2 \\ y = t^3 - t \end{cases}$$



Mathematica code:

```
ParametricPlot[{t^2, t^3-t}, {x, -2, 2}]
```