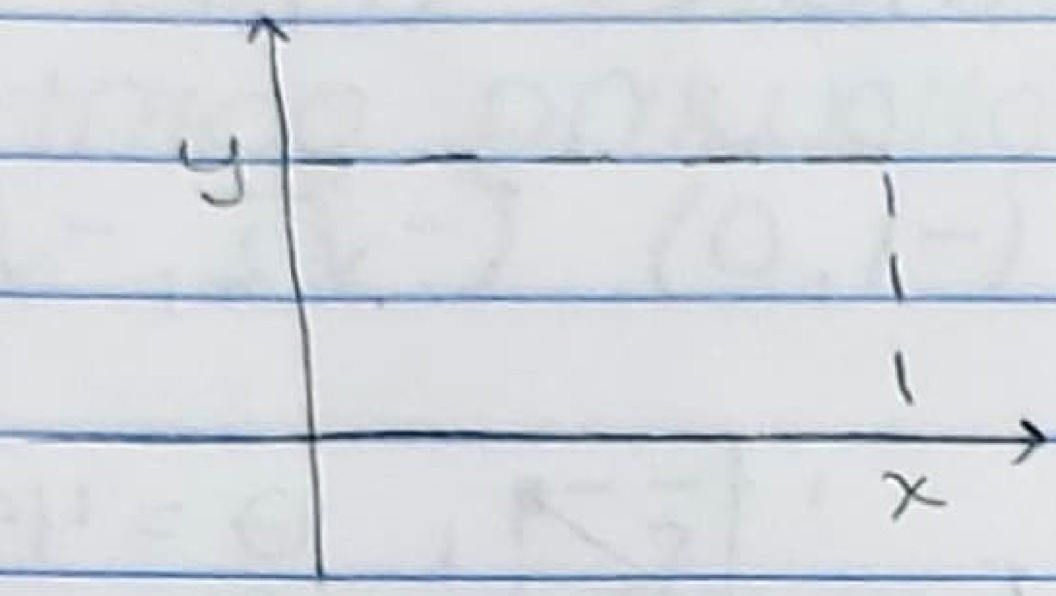
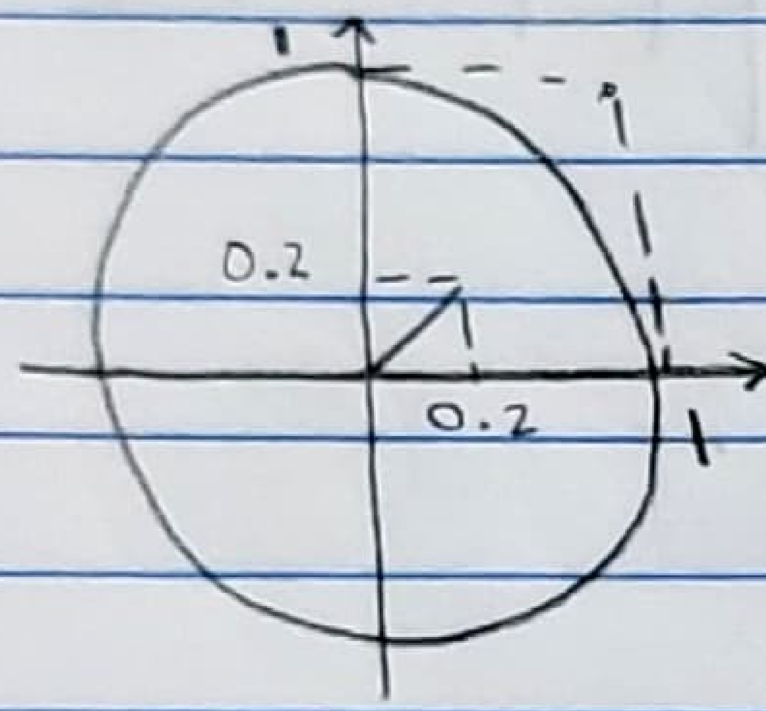
Polar Coordinates

5/22/23



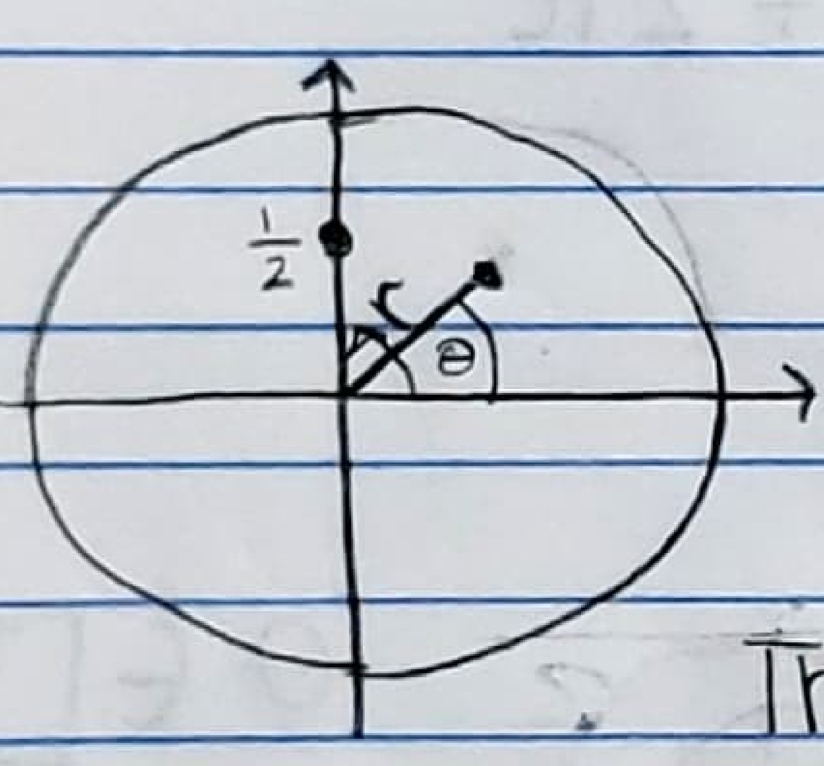
(x, y) - gives a unquie position on the plane



$(1, 1)$

$(0.2, 0.2)$

(x, y) in the circle if $x^2 + y^2 \leq 1$

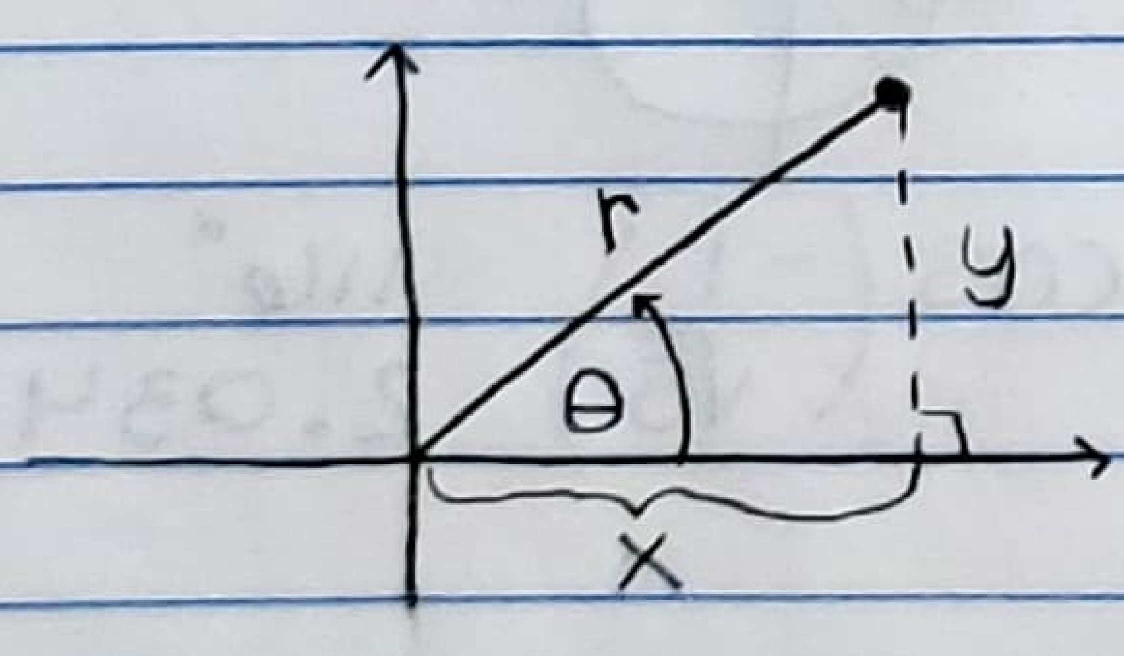


(r, θ) determine a unqiue point

$r = \frac{1}{2}$ $\quad (r, \theta)$: polar coordinate

$\theta = \frac{\pi}{2}$ $\quad (x, y)$: cartesian coordinate

The point is in the circle iff $0 \leq r \leq 1$



$(r, \theta) \leftrightarrow (x, y)$

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

$$r = \sqrt{x^2 + y^2}$$

$$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}}$$

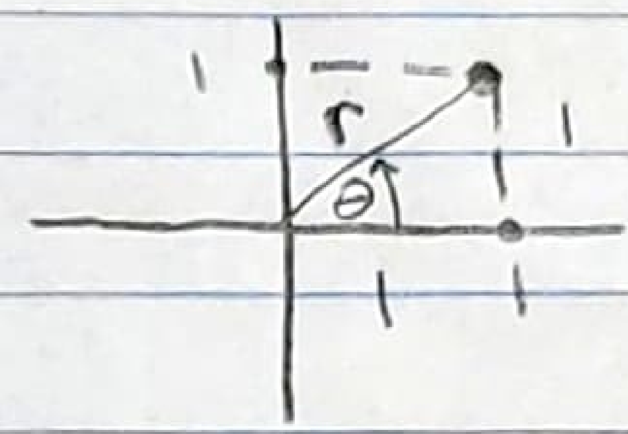
$$\sin\theta = \frac{y}{r} = \frac{y}{\sqrt{x^2+y^2}}$$

ex. Find the polar coordinates (r, θ)
$\theta \in [0, 2\pi]$ of the following points
$(x, y) = (1, 1) \quad (2, -1) \quad (-1, 0) \quad (-\sqrt{2}, -\sqrt{2})$

$(1, 1)$

$r = \sqrt{1+1} = \sqrt{2}$

$(r, \theta) \to (\sqrt{2}, \pi/4)$

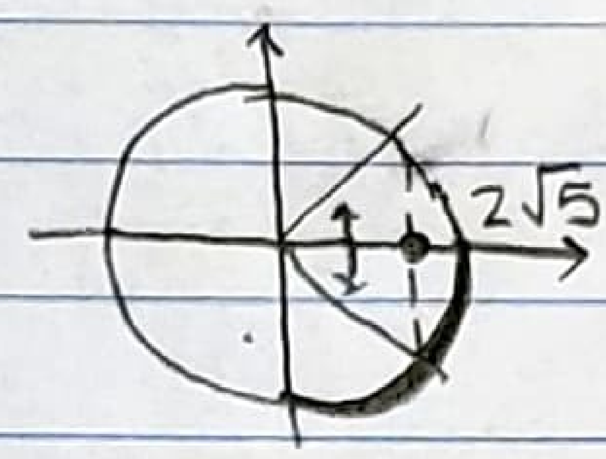


$\theta = 45^\circ$

$\pi/4$

$(2, -1)$

$r = \sqrt{4+1} = \sqrt{5}$

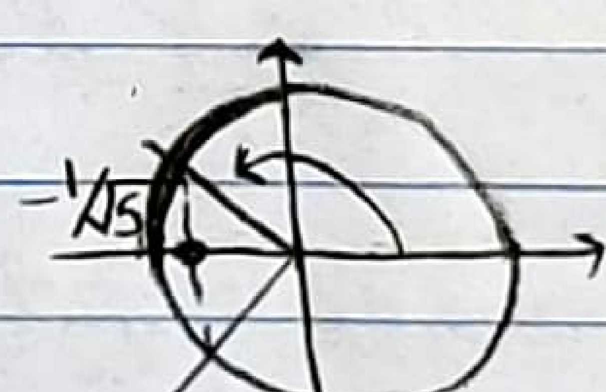


$\cos\theta = \dfrac{2}{\sqrt{5}} = -\arccos\left(\dfrac{2}{\sqrt{5}}\right) + 2\pi$

$\sin\theta = \dfrac{-1}{\sqrt{5}} \quad \arcsin\left(\dfrac{-1}{\sqrt{5}}\right)$

$(-1, 2)$

$r = \sqrt{5}$



$\theta \in [\pi/2, \pi]$

$\cos\theta = \dfrac{-1}{\sqrt{5}} \quad \theta = \arccos\left(\dfrac{-1}{\sqrt{5}}\right) \quad 116^\circ \quad 2.034$

$\sin\theta = \dfrac{2}{\sqrt{5}}$

arccos has range $[0, \pi]$

arcsin has range $[-\pi/2, \pi/2]$