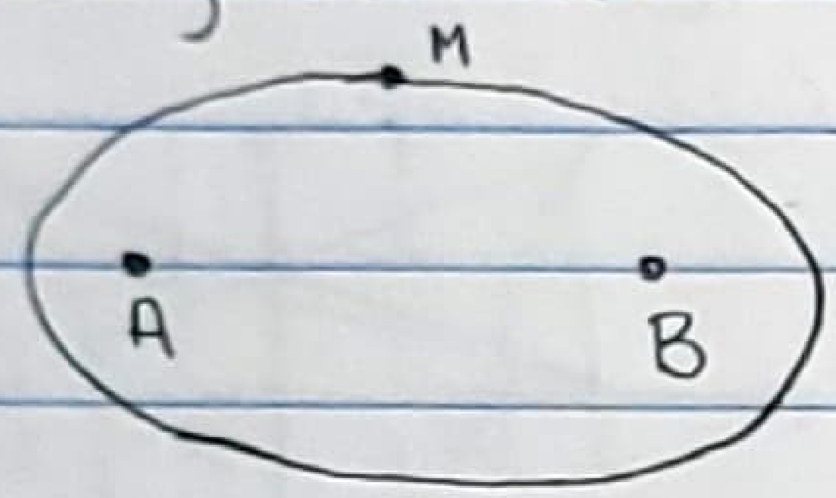
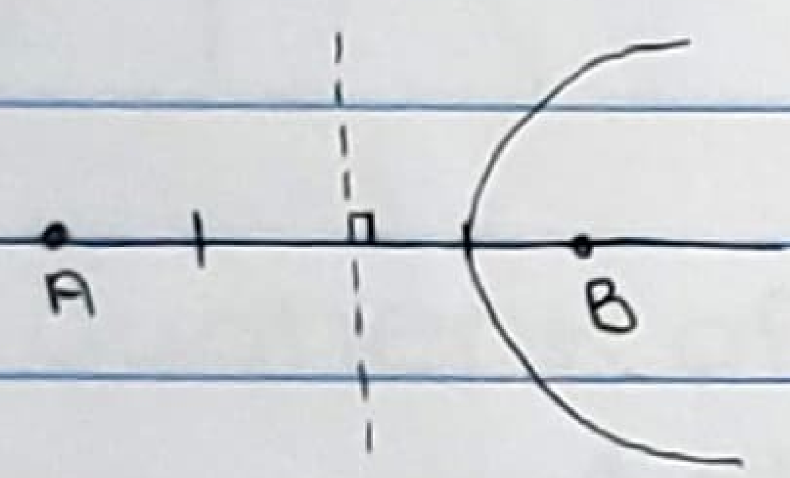
6/1/23

Conic Sections
↳ ellipse
↳ parabola
↳ hyperbola

* Using Distance to Foci:

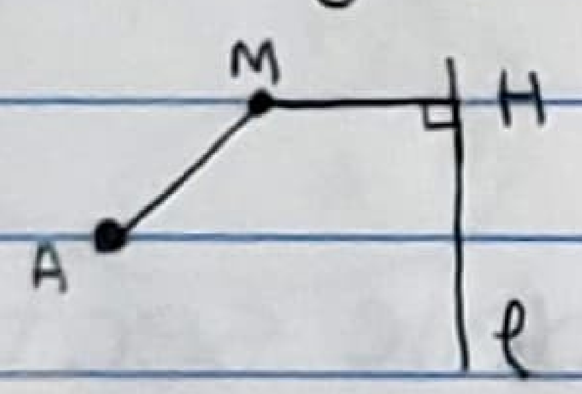


$AM + BM = \text{constant}$



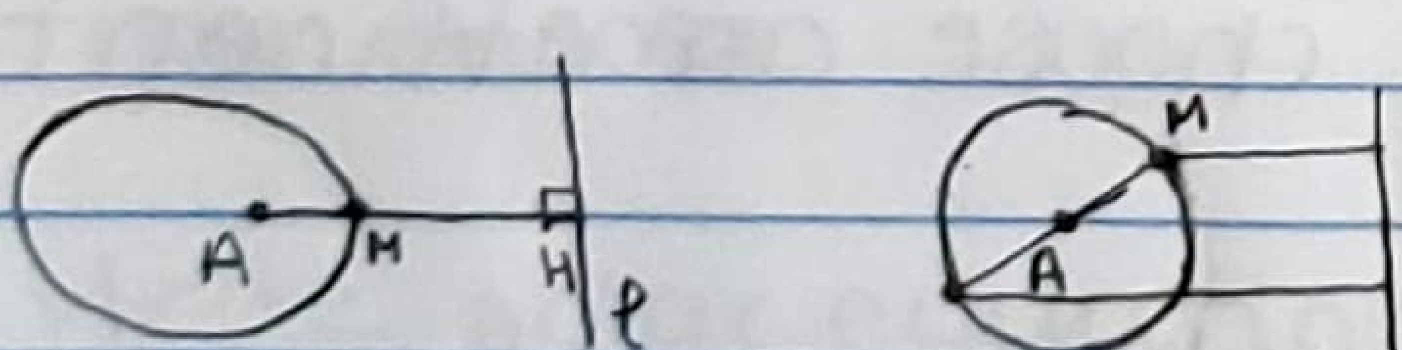
$AM - BM = \text{constant}$
* Hyperbola

* Using Focus and Directrix



$$\frac{MA}{MH} = e$$

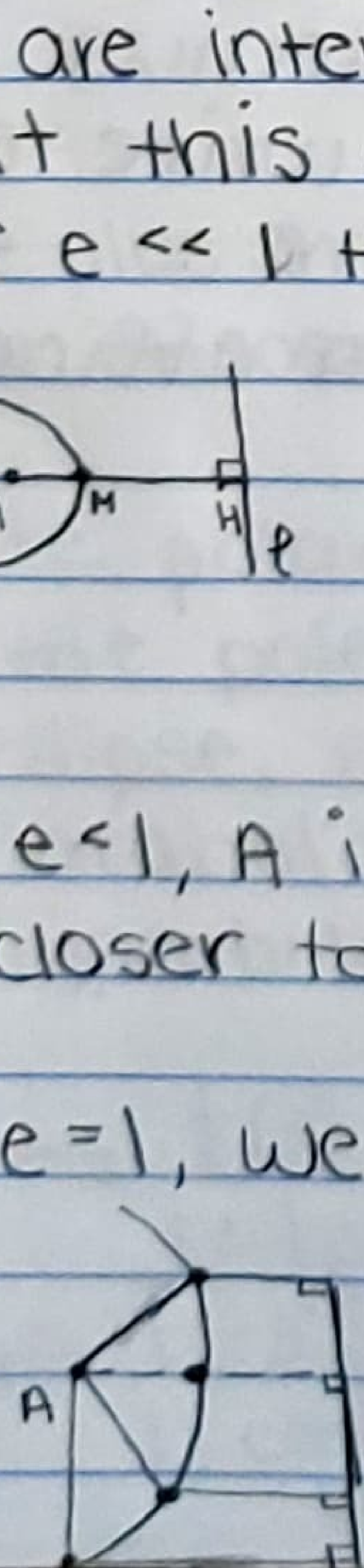
We are interested in set of points M such that this quotient is a constant $e > 0$.
If $e \ll 1$ then M has to be close to A



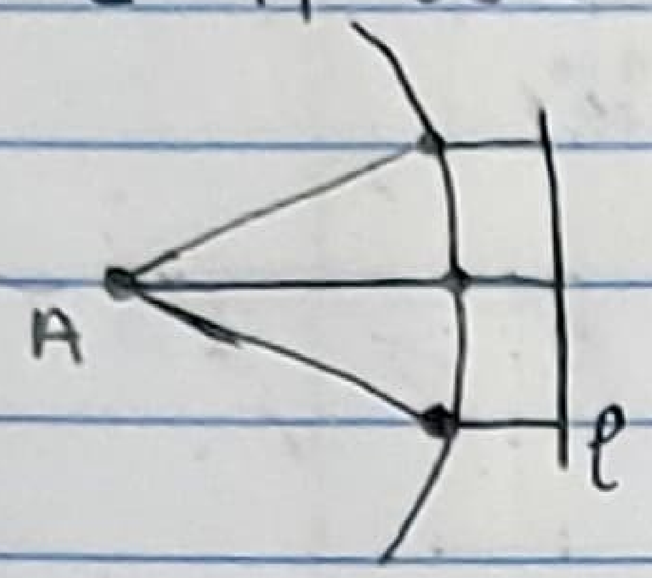
Can this ellipse be a circle?
No, because $MA = \text{const}$ but $MH \neq \text{const}$, so $\frac{MA}{MH} \neq \text{const}$

If $e < 1$, A is the focus of the ellipse that is closer to the line.

If $e = 1$, we get a parabola



If $e>1$, we get a hyperbola



A: focus
ℓ: directrix
e: eccentricity
$e<1$: ellipse
$e=1$: parabola
$e>1$: hyperbola

* Equation of Conic Section in Cartesian Coordinates

So far, we have described conic sections using geometry. To do calculus on those curves, we need to write an equation for them.
We will carefully choose an efficent coord system.