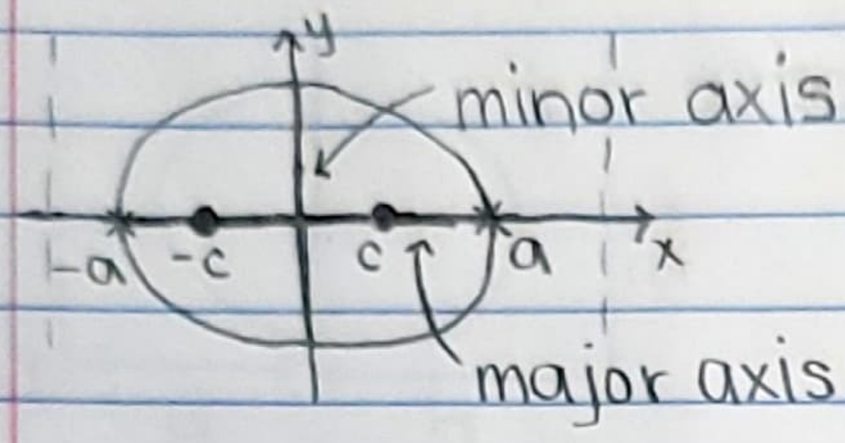
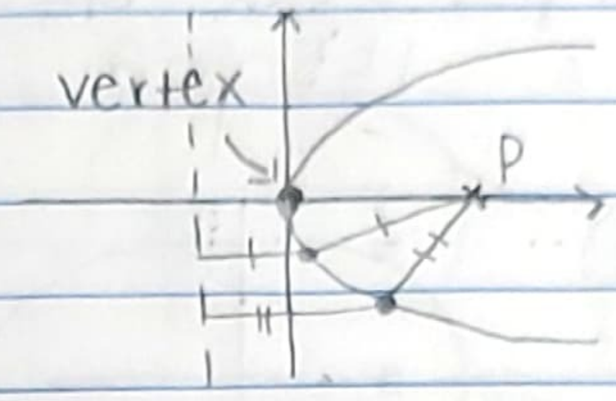
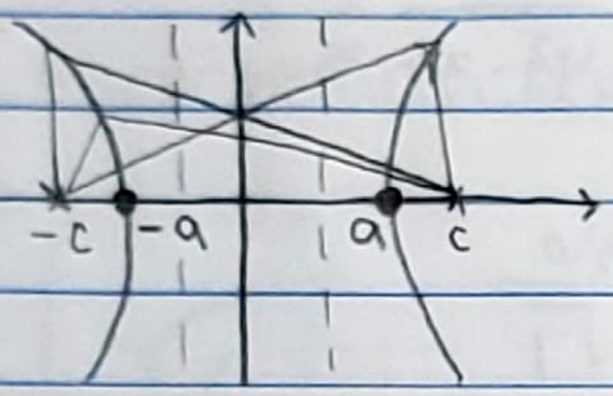
Equation of Conic Section

6/2/23

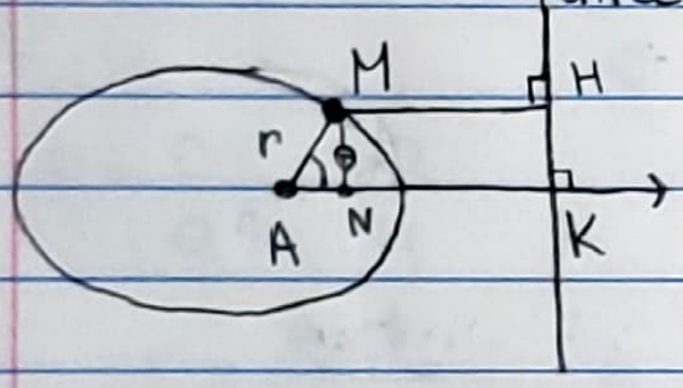






* 2 directrix for an ellipse
* 1 directrix for a parabola
* 2 directrix for a hyperbola

e = quotient of the difference



$$e = \frac{MH}{MA} = \frac{r}{NK} = \frac{r}{AK - AN} = \frac{r}{d - r\cos\theta}$$

$$e(d - r\cos\theta) = r \rightarrow ed - er\cos\theta = r$$

$$r = \frac{ed}{1 + e\cos\theta}$$

*the plus or minus depends on what directrix you choose

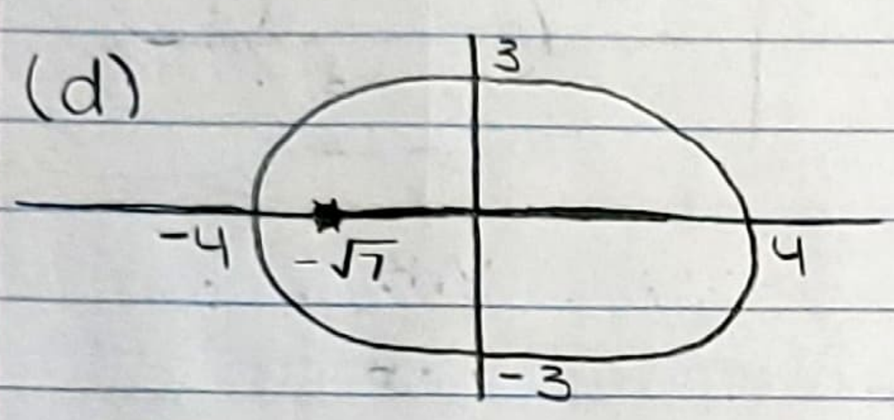
ex. Write polar equation of a conic with focus at the pole and with the following info:
(a) ellipse, eccentricity $= \frac{1}{3}$, directrix $x = 3$
(b) parabola, directrix $x = -3$
(c) hyperbola, eccentricity $= 5$, directrix $x = 3$

(a) $r = \frac{\frac{1}{3}(3)}{1 \pm \frac{1}{3}\cos\theta} = \frac{1}{1 \pm \frac{1}{3}\cos\theta}$

(b) $r = \frac{1(-3)}{1 \pm \cos\theta} = \frac{-3}{1 \pm \cos\theta}$

(c) $r = \frac{5(3)}{1 \pm 5\cos\theta} = \frac{15}{1 \pm 5\cos\theta}$

(d)



$c = \sqrt{4^2 - 3^2} = \sqrt{7}$

$e = \frac{c}{a} = \frac{\sqrt{7}}{4}$ $\quad d = \frac{a^2}{c} = \frac{16}{\sqrt{7}}$

$r = \frac{\frac{\sqrt{7}}{4}\left(\frac{16}{\sqrt{7}}\right)}{1 - \left(\frac{\sqrt{7}}{4}\right)\cos\theta} = \frac{4}{1 - \frac{\sqrt{7}}{4}\cos\theta}$ $\quad \theta = 0$