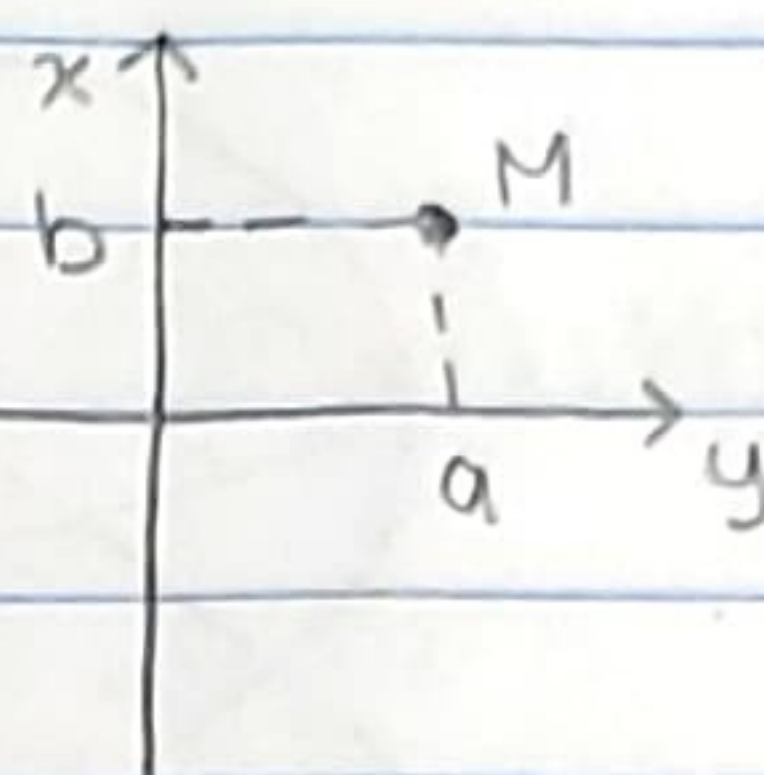
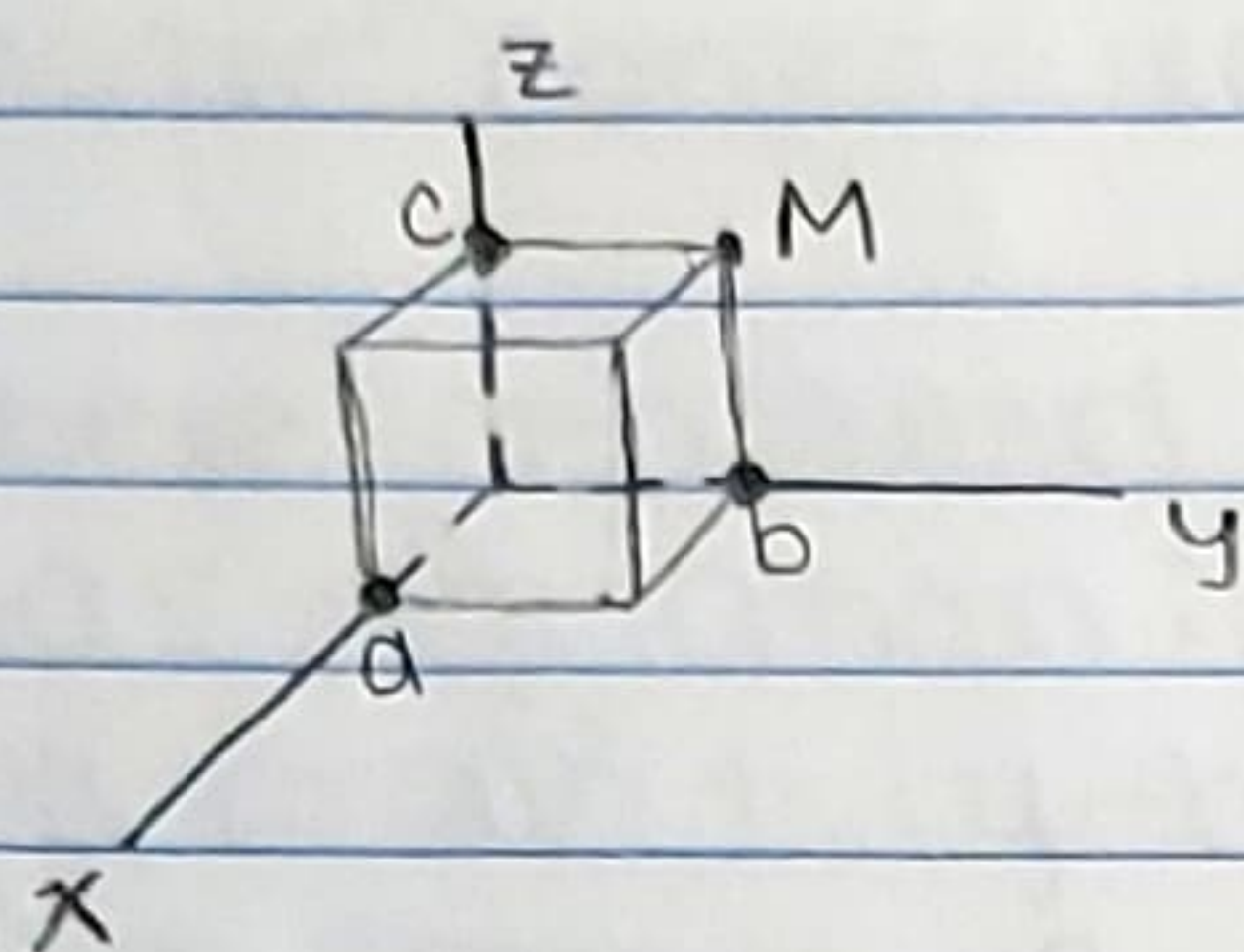
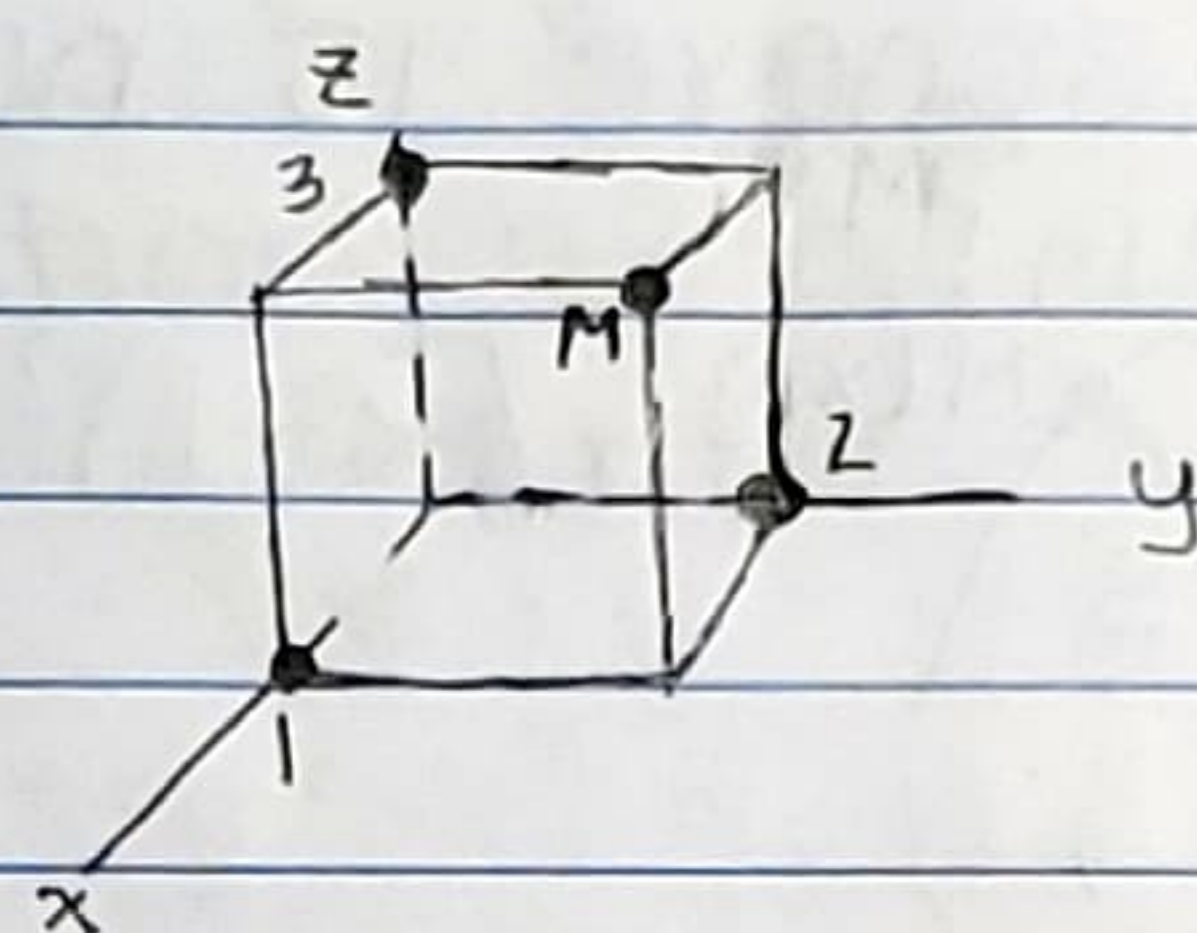


Three-dimensional Space

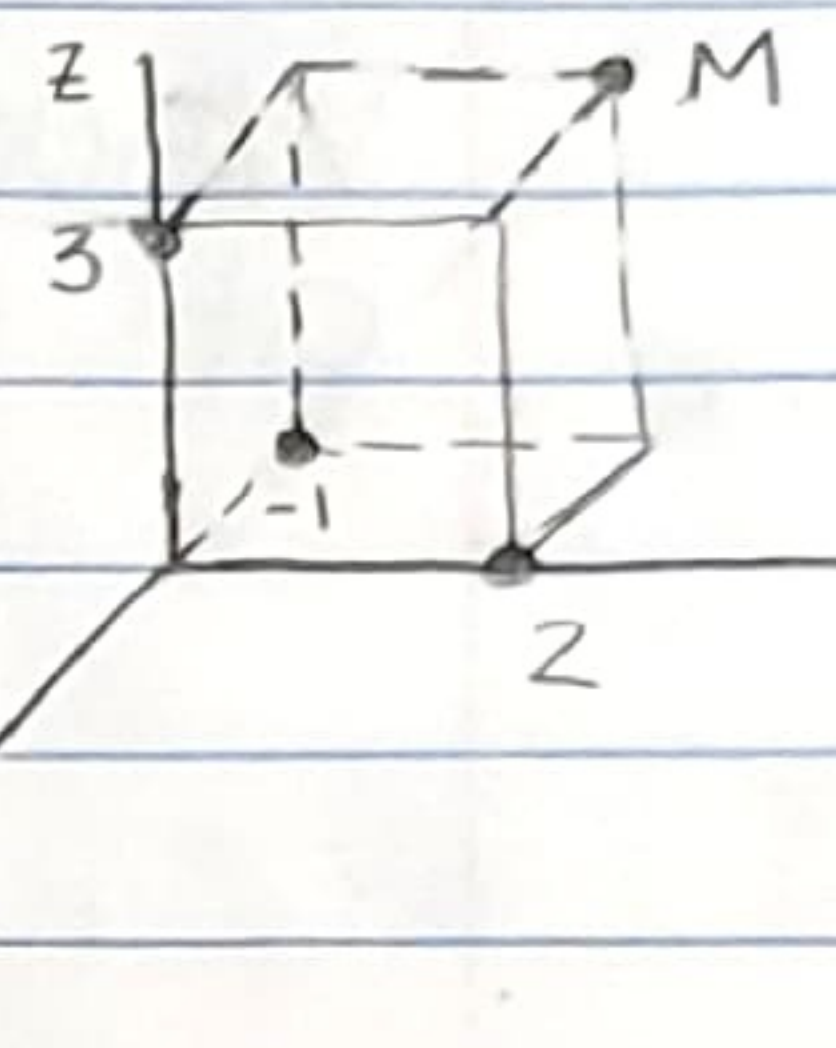
6/6/23



$M(1, 2, 3)$



$M(-1, 2, 3)$



$\left. \begin{array}{l} xy\text{-plane.} \\ yz\text{-plane} \\ zx\text{-plane} \end{array} \right\} \text{coordinate planes}$

The collection of point M such that $OM = \text{constant}$ is a sphere centered at the origin.

Eq of sphere: $M(x, y, z)$

$$OM = \sqrt{x^2 + y^2 + z^2} = r \rightarrow x^2 + y^2 + z^2 = r^2$$

The equation of the sphere centered at the origin with radius r .

ex. Which of the following points lie on, inside, outside the sphere centered at the origin with radius 3?

A $(-1, 2, 2)$ B $(1, -2, 1)$ C $(2, 1, -3)$

$$1 + 4 + 4 = 9$$

$$1 + 4 + 1 = 6$$

$$4 + 1 + 9 = 14$$

$$9 = 9$$

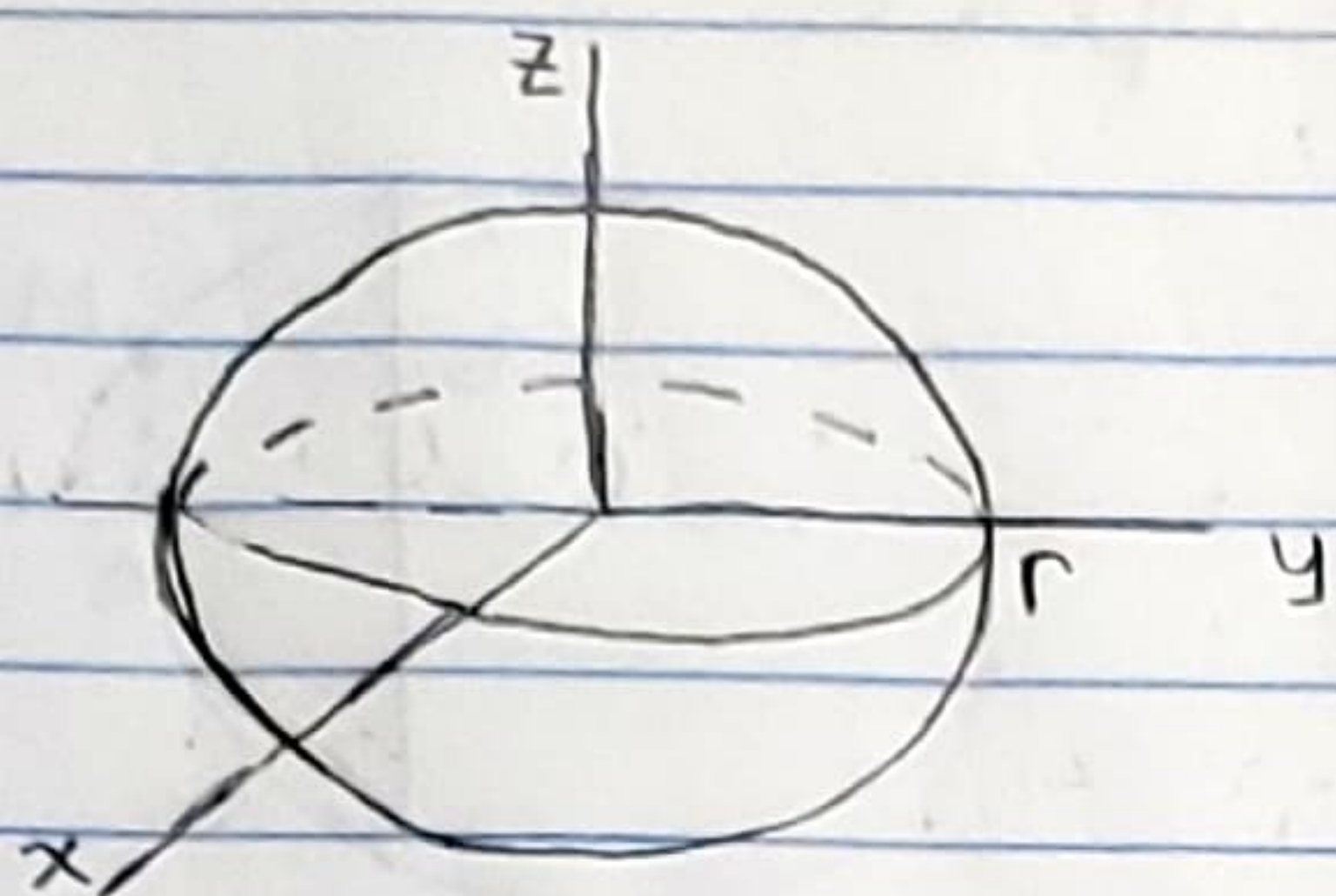
$$6 < 9$$

$$14 > 9$$

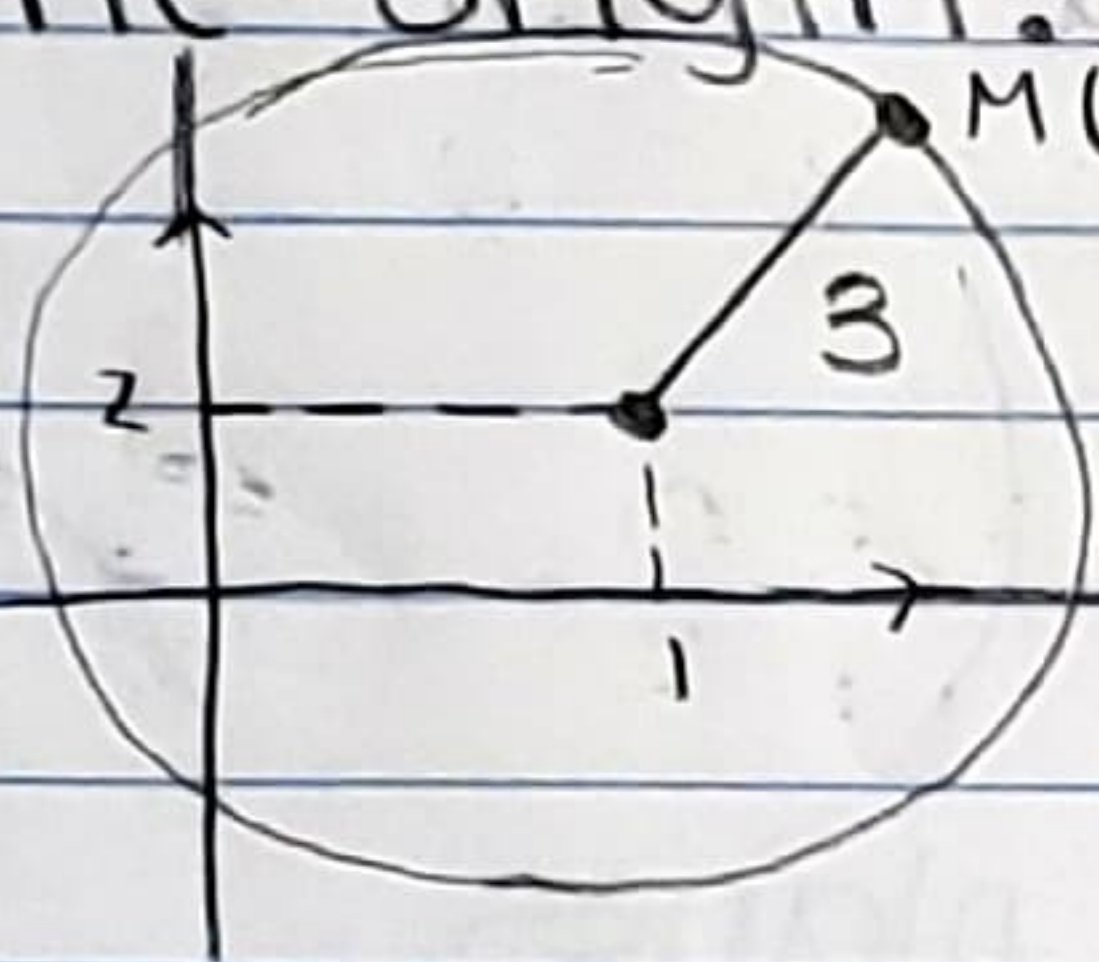
A lies on.

B lies inside.

C lies outside



What if the sphere is not centered at the origin?



$$(x-1)^2 + (y-2)^2 = 3^2$$

The distance between $P(a, b, c)$ and $Q(d, e, f)$ is given by

$$PQ = \sqrt{(a-d)^2 + (b-e)^2 + (c-f)^2}$$

ex. The sphere centered at $(1, -1, -2)$ with radius 5 is:

$$AM^2 = 5^2$$

$$(x-1)^2 + (y+1)^2 + (z+2)^2 = 25$$

ex. $x^2 + y^2 + z^2 - 2x + 4y + 2z = 10$
 $(x^2 - 2x) + (y^2 + 4y) + (z^2 + 2z) = 10$