Vectors 6/8/23

* poincare' conjecture

A sphere centered at (a, b, c) with radius r has equation:

$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

a sphere is a quadratic surface.

ex. $2x^2+2y^2+2z^2=5x+2y-z$
is also equation of a sphere.

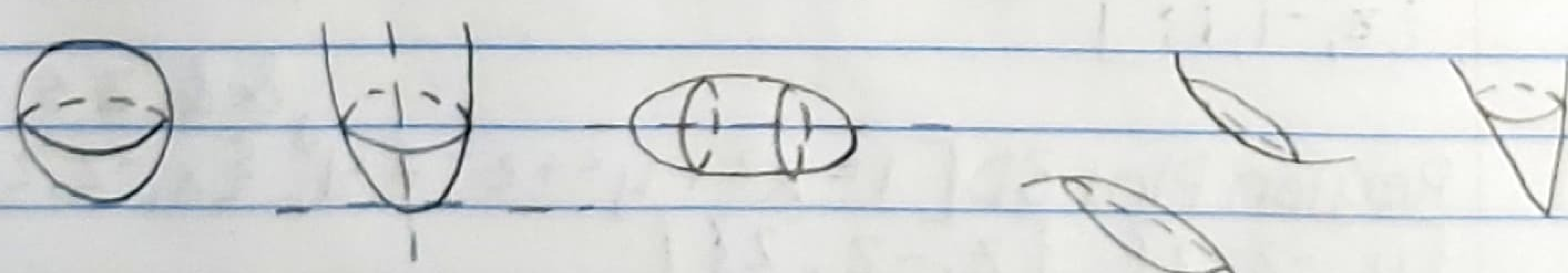
$$2x^2+2y^2+2z^2-5x+2y-z=0$$
$$x^2+y^2+z^2-\frac{5}{2}x+y-\frac{1}{2}z=0$$
$$x^2-\frac{5}{2}x+y^2-y+z^2-\frac{1}{2}z=0 \qquad +\frac{25}{16}+\frac{1}{4}+\frac{1}{16}=\frac{30}{16}=\frac{15}{8}$$
$$\frac{4}{16}$$
$$\left(x-\frac{5}{4}\right)^2+\left(y-\frac{1}{2}\right)^2+\left(z+\frac{1}{4}\right)^2=\frac{15}{8}$$

center is $\left(\frac{5}{4},\frac{1}{2},-\frac{1}{4}\right)$ with radius $\sqrt{\frac{15}{8}}$

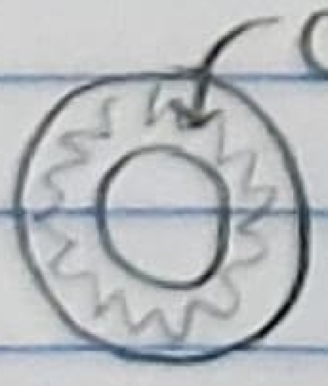
⇒ A general quadratic surface has the equation:

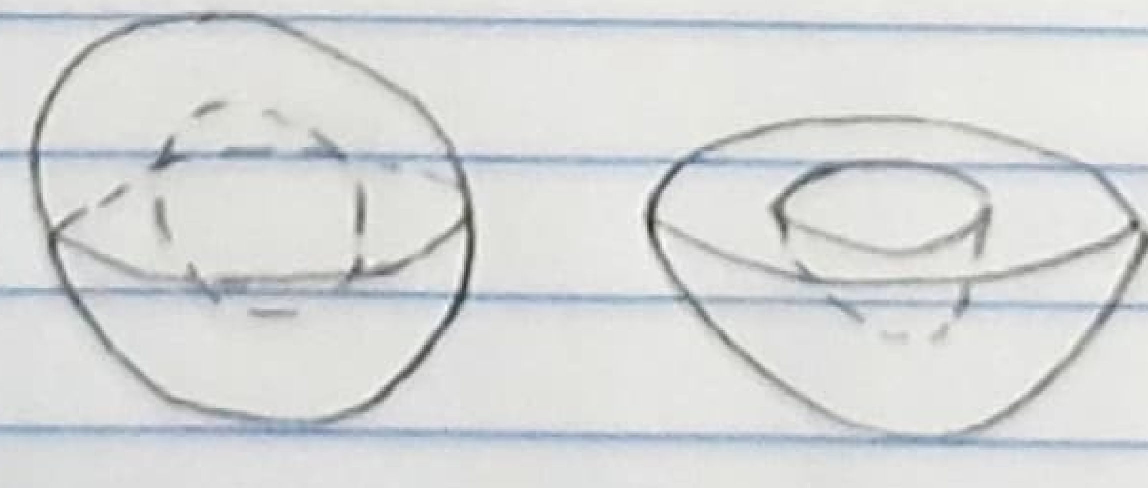
$$Ax^2+By^2+Cz^2+Dxy+Eyz+Fzx+Gx+Hy+Iz+J=0$$

* Special cases: sphere, parabolord, ellipsiod, hyperbolbid, cone, cylinder, ...

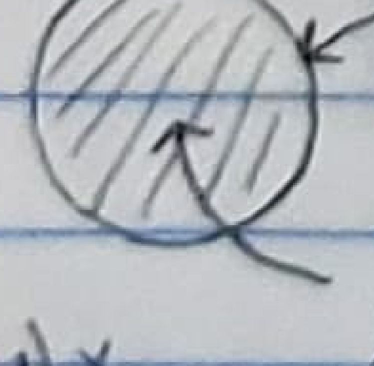


annulus





Shell



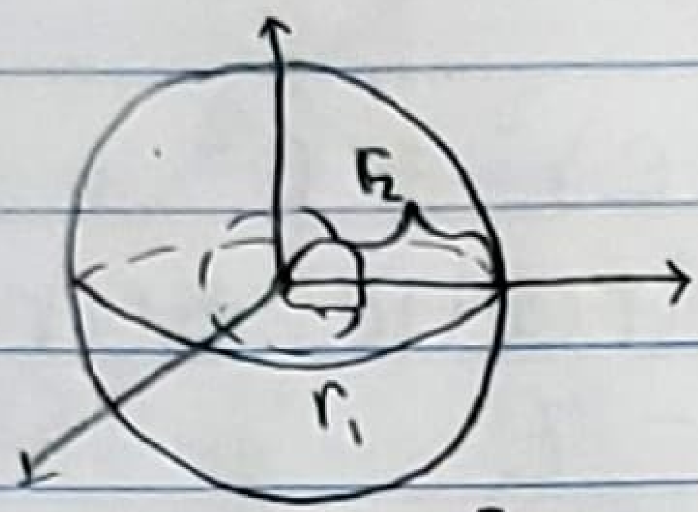
$x^2+y^2=1$

$x^2+y^2<1$

$(x,y)\times$ $x^2+y^2>1$

The solid sphere (ball) centered at (a, b, c) with radius r:

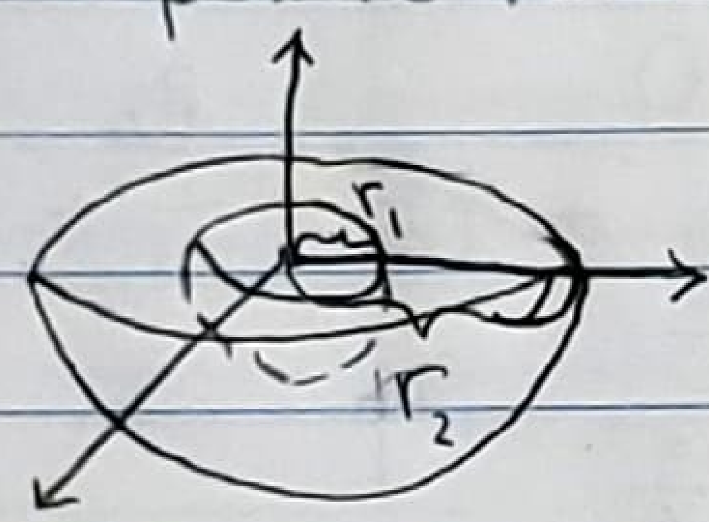
$(x-a)^2 + (y-b)^2 + (z-c)^2 \le r^2$



The shell with inner radius r_1, outer radius r_2:

$r_1^2 \le x^2 + y^2 + z^2 \le r_2^2$

points that satisfy inequality lie inside



$$\begin{cases} r_1^2 \le x^2 + y^2 + z^2 \le r_2^2 \\ z \le 0 \end{cases}$$

Mathematica Commands

Sphere →
ContourPlot3D[x² + y² + z² == 1, {x, -1, 1}, {y, -1, 1}, {z, -1, 1}]

&& z ≤ 0

RegionPlot3D[1 ≤ x² + y² + z² ≤ 4, {x, -2, 2} {y, -2, 2}, {z, -2, 2}]