Lecture 4

Thursday, April 6, 2023 10:45 PM

* Questions ....

How to find the limit?

If $a_n = f(n)$ and $\lim_{x\to\infty} f(x) = L$ then $\lim a_n = L$.

How to show that the limit doesn't exist?

(1) If there are two subsequences that converge to two different limits then the sequence $\{a_n\}$ diverges.

(2) If there is a subsequence that converges to $\pm\infty$ then the sequence $\{a_n\}$ diverges.

Ex

$$a_n = (-1)^n \frac{n}{n+1}$$

$$a_{2k} = \frac{2k}{2k+1} \longrightarrow 1 \quad \text{as } k\to\infty$$

$$a_{2k+1} = -\frac{2k+1}{2k+2} \longrightarrow -1 \quad \text{as } k\to\infty$$

The two subsequences $\{a_{2k}\}$ and $\{a_{2k+1}\}$ converge to two different limits. Therefore, $\{a_n\}$ diverges.

Ex Find the limit of $a_n = (-1)^{n+1} \dfrac{n}{n+\sqrt{n}}$

* n even

$$a_n = -\frac{n}{n+\sqrt{n}} = -\frac{1}{1+n^{-1/2}} \longrightarrow -1 \quad \text{as } n\to\infty$$

* n odd

$$a_n = \frac{n}{n+\sqrt{n}} = \frac{1}{1+n^{-1/2}} \longrightarrow 1 \quad \text{as } n\to\infty$$

Therefore, $\{a_n\}$ diverges.

Ex

$$a_n = (-1)^n \underbrace{\sin\left(\frac{\pi}{6}+n\pi\right)}_{=(-1)^n \sin\left(\frac{\pi}{6}\right)} = \sin\frac{\pi}{6} = \frac{1}{2}$$

$\{a_n\}$ is a constant sequence and $\lim a_n = \frac{1}{2}$.

Ex

$$a_n = (-1)^n \frac{n}{\underbrace{n^2-1}_{\text{goes to } 0 \text{ as } n\to\infty}}$$

$$|a_n| = \frac{n}{n^2-1} \to 0 \quad \text{as } n\to\infty$$

$$|a_n - 0| \longrightarrow 0 \quad \text{as } n\to\infty$$

Therefore, $\lim a_n = 0$

Squeeze theorem

If $b_n \le a_n \le c_n$ and $\lim b_n = \lim c_n = L$ then $\lim a_n = L$.

Ex $\lim\limits_{n\to\infty} \frac{\sin n}{n} = ?$

$$-\frac{1}{n} \le \frac{\sin n}{n} \le \frac{1}{n} \qquad \text{Thus, } \lim_{n\to\infty} \frac{\sin n}{n} = 0$$

0

$\{a_n\}$ is said to be increasing if $a_n \le a_{n+1}$ for all n.

$\{a_n\}$ is said to be decreasing if $a_n \geq a_{n+1}$ for all n.

$\{a_n\}$ is said to be monotonic if it is increasing or decreasing

$\{a_n\}$ is said to be bounded from above if $a_n \le M$ for all n

$\{a_n\}$ is said to be bounded from below if $a_n \geq m$ for all n.

$\{a_n\}$ is said to be bounded if it is bounded from above or below.

Ex $\left\{\frac{\sin n}{n}\right\}$ is bounded because

$$-1 \le -\frac{1}{n} \le \frac{\sin n}{n} \le \frac{1}{n} \le 1 \quad \text{for all } n.$$