Lecture 5

Monday, April 10, 2023 10:19 AM

* Questions....

How to show that a sequence is increasing?

$$a_n = \frac{2n-3}{3n+4} = f(n) \quad \text{where} \quad f(x) = \frac{2x-3}{3x+4}$$

If f is an increasing function then $\{a_n\}$ is also an increasing sequence.

$$f'(x) = \frac{2(3x+4) - 3(2x-3)}{(3x+4)^2} = \frac{17}{(3x+4)^2} > 0$$

Thus, f is an increasing function

<u>Series</u>

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \cdots$$

The n'th partial sum of this series is defined as

$$S_n = a_1 + a_2 + \cdots + a_n$$

<u>Definition</u>:

$$\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n$$

If the limit exists then the series is said to <u>converge</u>.

Otherwise, the series is said to <u>diverge</u>.

<u>Ex</u> $1+2+3+4+\cdots$ is a divergent series

$1-1+1-1+1-1+1-1+\ldots$ is also a divergent series.

Why?

$$S_n = \begin{cases} 1 & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}$$

$$\lim_{\substack{n\to\infty \\ n \text{ odd}}} S_n = 1 \neq \lim_{\substack{n\to\infty \\ n \text{ even}}} S_n = 0.$$

There are special types of series which we will look into detail.

<u>Geometric series</u>

This is the kind of series in which the ratio $\frac{a_{n+1}}{a_n}$ is a constant.

The number $r = \frac{a_{n+1}}{a_n}$ is called the <u>common ratio</u>.

<u>Ex</u>

$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\ldots$ is a geometric series with common ratio $\frac{1}{2}$.

$\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^n$ is also a geometric series because

$$\frac{a_{n+1}}{a_n} = \frac{\left(\frac{2}{3}\right)^{n+1}}{\left(\frac{2}{3}\right)^n} = \frac{2}{3}$$

$\sum_{n=0}^{\infty}(-1)^{n+1}$ is a geometric series because

$$\frac{a_{n+1}}{a_n} = \frac{(-1)^{n+2}}{(-1)^{n+1}} = -1$$

$\sum_{n=1}^{\infty} \frac{1}{n}$ is not a geometric series because

$$\frac{a_{n+1}}{a_n} = \frac{\frac{1}{n+1}}{\frac{1}{n}} = \frac{n}{n+1}$$ depends on n.