# Lecture 6

Tuesday, April 11, 2023 8:15 AM

* Questions...

Recall geometric series: $\sum_{n=1}^{\infty} a_n$, where $\frac{a_{n+1}}{a_n} = r$ (constant)

$$a + ar + ar^2 + ar^3 + \cdots$$

Theorem:

$$a + ar + ar^2 + \cdots + ar^{n-1} = a(1 + r + r^2 + \cdots + r^{n-1}) = a\frac{1-r^n}{1-r}$$

* Observation:

If $-1 < r < 1$ then $r^n \to 0$ as $n \to \infty$ and $a + ar + ar^2 + \cdots = \frac{a}{1-r}$.

If $r \le -1$ or $r \ge 1$ then $r^n$ doesn't have a limit, and the series diverges

Ex Is the series $\sum_{n=1}^{\infty} 3^{n+1} 2^{-2n}$ convergent? If so, find its value.

$$\left.\begin{array}{l} a_n = 3^{n+1} 2^{-2n} \\ a_{n+1} = 3^{n+2} 2^{-2n-2} \end{array}\right\} \quad \frac{a_{n+1}}{a_n} = \frac{3^{n+2} 2^{-2n-2}}{3^{n+1} 2^{-2n}} = \frac{3 \cdot 2^{-2}}{1} = \frac{3}{4}$$

This is a geometric series with common ratio $r = \frac{3}{4} \in (-1, 1)$.

Therefore, the series converges. The first term of this series is $3^{1+1} 2^{-2(1)} = \frac{9}{4}$

$$\frac{9}{4} + \frac{9}{4}\left(\frac{3}{4}\right) + \frac{9}{4}\left(\frac{3}{4}\right)^2 + \frac{9}{4}\left(\frac{3}{4}\right)^3 + \cdots = \frac{9}{4}\left[1 + \frac{3}{4} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \cdots\right]$$

$$= \frac{9}{4} \frac{1}{1-\frac{3}{4}} = 9.$$

Ex

$$\sum_{n=2}^{\infty} (-1)^{n+1} 2^{n+2} 5^{-\frac{n}{2}}$$

Does it converge? what is its value?

On mathematica, we use the command Sum.