Polar equation of an ellipse

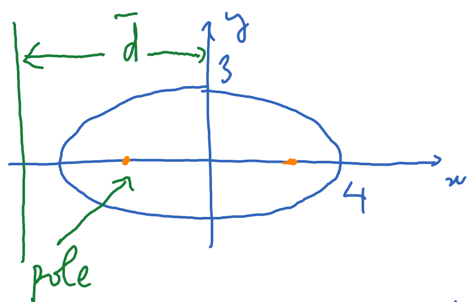
Saturday, June 3, 2023 11:49 PM

Recall that the polar equation of an ellipse is

$$r = \frac{ed}{1 \pm e\cos\theta}$$

Here, the pole is at one of the foci of the ellipse and d is the distance from the pole to the directrix.

Ex Find a polar equation of the following ellipse.



$a = 4$, $b = 3$

$\rightsquigarrow c = \sqrt{a^2 - b^2} = \sqrt{7} \rightsquigarrow e = \frac{c}{a} = \frac{\sqrt{7}}{4}$

The distance from the origin to the directrix is $\bar{d} = \frac{a^2}{c} = \frac{16}{\sqrt{7}}$

Thus, the distance from the pole $(-\sqrt{7}, 0)$ to the directrix is

$$d = \bar{d} - c = \frac{16}{\sqrt{7}} - \sqrt{7} = \frac{9}{\sqrt{7}}$$

Therefore, the polar equation of the ellipse is

$$r = \frac{ed}{1 \pm e\cos\theta} = \frac{\frac{\sqrt{7}}{4} \frac{9}{\sqrt{7}}}{1 \pm \frac{\sqrt{7}}{4}\cos\theta} = \frac{\frac{9}{4}}{1 \pm \frac{\sqrt{7}}{4}\cos\theta} = \frac{9}{4 \pm \sqrt{7}\cos\theta}$$

To know which sign to take, we check $\theta = 0$.

When $\theta = 0$, the point on the ellipse is at $(4, 0)$. The distance from it to the pole is $r = 4 + \sqrt{7}$.

Also,

$$r = \frac{9}{4 \pm \sqrt{7}\cos 0} = \frac{9}{4 \pm \sqrt{7}}$$

For this to be equal to $4 + \sqrt{7}$, we pick the minus sign. Therefore,

$$r = \frac{9}{4 - \sqrt{7}\cos\theta}$$

* Note: the focus and the directrix go together. There are two foci and two directrices. They go in pair as in the below picture.