

Final exam: Some problems for review

The exam will be held at the regular classroom Loso 116 from Tuesday June 13 from 8 AM to 10 AM. The material covered is Section 12 (complex root only), 14, 18, 20, 21, 27 (using substitution is acceptable) of the textbook. It is a closed book exam. A 4" x 6" handwritten single-sided note card is allowed. A scientific calculator is allowed (*and you will need it!*) Graphing/ programmable/ transmittable calculators are not allowed.

You should review the homework problems, the examples given in the textbook and in the lectures. It is always a good idea to study for the exam with someone. The types of problems you may be asked on the exam include:

- Use the “characteristic equation” method to solve a homogeneous second order ODE.
- Use the *method of undetermined constants* to solve an inhomogeneous second order ODE.
- Use *variation of constants* to solve an inhomogeneous second order ODE.
- Use the power series technique to solve an ODE.
- Use Euler’s method to solve approximately the solution of a first order ODE.
- Solve a simple 2-by-2 linear system of ODEs.

Additional problems to practice:

- 1) Find the general solution to the equation $x'' + 4x' + 5 = 0$.
- 2) Find the general solution to the equation $x'' + x' - 2x = 2e^{-2t} + t$.
- 3) Find the general solution to the equation $x'' - 3x' + 2x = \sin(e^{-t})$.
- 4) Consider to equation $y'' - xy' + y = e^{2x}$ with initial conditions $y(0) = 1, y'(0) = 2$.
 - (a) If we approximate $y \approx a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$. What would a_0, a_1, a_2, a_3, a_4 be?
 - (b) Express y as a power series $y = \sum_{n=0}^{\infty} a_nx^n$. Find a recursive formula for a_n 's.
- 5) Consider the equation

$$y' = \frac{1}{1 + xy}$$

with the initial condition $y(0) = 1$. Use Euler’s method to find approximately $y(0.1), y(0.2), y(0.3), y(0.4)$.

- 6) Solve the system of differential equations

$$\begin{aligned}x_1' &= x_1 + 2x_2 \\x_2' &= 3x_1 + 2x_2\end{aligned}$$

with the initial conditions $x_1(0) = -2$ and $x_2(0) = 3$.