Lecture 1

Thursday, March 30, 2023 4:20 PM

* Introduction

This course is built upon Calc I and Calc II. The tools we will use are derivative, integral, limit, and some other tools to be developed along this course.

Differential equation is an equation to be solved for some unknown. Unlike the equations you have seen before, the solution to a diff eq. is a function, not a number.

Ex

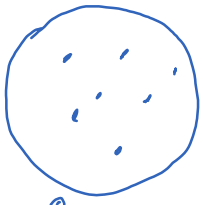

Plate containing
7 bacteria

These bacteria multiply in time. The more bacteria there are, the faster the population grow.

Current population is $x'(t)$, which is assumed to be proportional to the current population $x(t)$.

$x'(t) = \alpha x(t)$ ← This is a differential equation.

Convention: We will simply write x instead of $x(t)$, with the understanding that x is a dependent variable, and t is an independent variable.

We need to solve for x as a function of t from the equation

$$\begin{cases} x' = \alpha x \\ x(0) = 7 \end{cases}$$

The factor α depends on the setting, so let's take $\alpha = 2$.

$$\left(x' = 2x\right) \times dt$$

$$\underbrace{x'\,dt}_{dx} = 2x\,dt$$

$$\rightsquigarrow \quad \frac{dx}{x} = 2\,dt$$

$$\rightsquigarrow \quad \int \frac{dx}{x} = \int 2\,dt$$

$$\rightsquigarrow \quad \ln x = 2t + C$$

$$\rightsquigarrow \quad x = e^{2t+C} = e^{2t}\underbrace{e^{C}}_{k} = k e^{2t}$$

To determine k, we need the condition $x(0) = 7$.

Substitute $t=0$:

$$x(0) = ke^{0} \rightsquigarrow k = 7$$

Conclusion: $\boxed{x = 7e^{2t}}$

Differential equations arise naturally in real-life problem.