Lecture 11

Friday, April 21, 2023 2:40 PM

* Questions..

Carbon dating

^{12}C : normal isotope of carbon (stable)

^{13}C : 1% of carbon atoms

^{14}C radioactive isotope, very rare, decaying over time

In living things, the ratio $^{14}C : ^{12}C$ is about $1.35 : 10^{12}$.

After the living matter dies, ^{14}C starts decaying, but ^{12}C remains the same.

From analyzing the ratio $^{14}C : ^{12}C$, people can estimate the time between death and the presence.

From time t to $t+\Delta t$, a ^{14}C atom has a chance $k\Delta t$ of disappearing.

A collection of $N = N(t)$ ^{14}C atoms would become

$$N(t+\Delta t) = N(t) - (k\Delta t) N(t)$$

$$\rightsquigarrow \quad \frac{N(t+\Delta t) - N(t)}{\Delta t} = -k N(t)$$

Let $\Delta t \to 0$

$$N'(t) = -k N(t)$$

$$\rightsquigarrow \quad N = C_1 e^{-kt}$$

$$\rightsquigarrow \quad \frac{N(t)}{N(0)} = e^{-kt}$$

Half-life of ^{14}C is the time t^* it takes for the amount of ^{14}C to reduce by half.

$$\frac{1}{2} = \frac{N(t^*)}{N(0)} = e^{-kt^*}$$

$$\rightsquigarrow \ln\left(\frac{1}{2}\right) = -k t^*$$

$$\rightsquigarrow k = \frac{\ln 2}{t^*}$$

It is known that $t^* \approx 5700$ years, so $k = \dfrac{\ln 2}{5700}$.

$\underline{\underline{\text{Ex}}}$ While examining an artifact, scientists found that the ratio ^{14}C ^{12}C is only 91% of the usual ratio on living plants. When was the artifact made (from a living tree)?

$$91\% = \frac{\dfrac{N(t_2)}{\# ^{12}C}}{\dfrac{N(t_1)}{\# ^{12}C}} = \frac{N(t_2)}{N(t_1)} = \frac{C_1 e^{-kt_2}}{C_1 e^{-kt_1}} = e^{-k(t_2 - t_1)}$$

$$\rightsquigarrow \ln(0.91) = -k(t_2 - t_1)$$

$$\rightsquigarrow t_2 - t_1 = \frac{\ln(0.91)}{-k} \approx \frac{\ln(0.91)}{-\ln 2} 5700 \approx 775 \text{ years.}$$