

# Lecture 13

Tuesday, April 25, 2023 8:05 AM

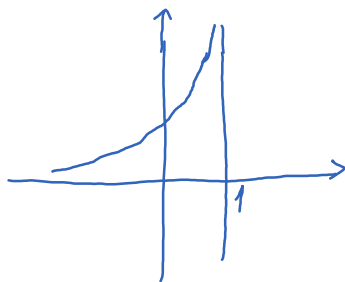
\* Questions...

Explain more on the theorem of existence and uniqueness, including the example of the Navier-Stokes equations.

## Examples

(1)  $y' = y^2, y(0) = 1$

$$y = \frac{1}{1-x}$$

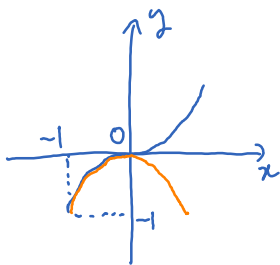


Solution fails to exist beyond  $x=1$ .

Here the function  $f(x,y) = y^2$  is continuous everywhere. So is  $\frac{\partial f}{\partial y} = 2y$ .

However, the solution doesn't exist for every  $x$ .

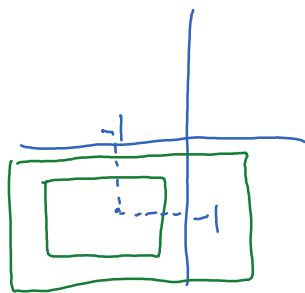
(2)  $y' = 3xy^{1/3}, y(1) = -1$



Here  $f(x,y) = 3xy^{1/3}$ .

$$\frac{\partial f}{\partial y} = 3x \cdot \frac{1}{3} y^{-2/3} = \frac{x}{y^{2/3}}$$

$\frac{\partial f}{\partial y}$  fails to be continuous when  $y = 0$ .



There are many ways to choose the rectangle  $R$ , but none of them can contain any part of the  $x$ -axis.

After  $y$  reaches 0, the solution fails to be unique.

\* Food for thought:

$$y' = y^{1/3}, \quad y(0) = 0$$

This initial value problem has a solution  $y \equiv 0$  (constant function).

Does it have another solution? Extra credit for correct answer.