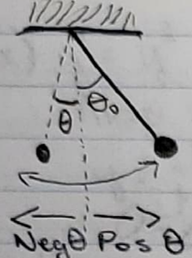
4/27



Given time t, where is the pendulum

$\theta = \theta(t)$

Wolonet!
Autonomus
Nonlinear (cuz sin)
2^{nd} order ODE $\rightarrow$ $\theta'' + \frac{g}{l}\sin\theta = 0$ $\rightarrow$ g = gravity constant
l = length of the string

2^{nd} order from accelleration
($F=ma$)

Must have an initial condition $\theta(0)=a$
AND the initial velocity $\theta'(0)=b$

How to solve for θ?
Attempt: $\theta'' + \frac{g}{l}\sin\theta = 0 \Rightarrow \theta'' + C\sin\theta = 0$
$\left(\frac{\theta''}{\sin\theta} = -c\right) dt \Rightarrow \frac{d(\theta')}{\sin\theta} = C\,dt$ $\Leftarrow$ Can't Integrate
Stuck!

θ in Radians New Method:
If $\theta \approx 0°$
$\sin\theta \approx \theta$ $\left(Ex: 20° = \frac{20}{180}\pi \approx .349 \Rightarrow \sin(.349) = .342\right)$
$\theta'' + c\sin\theta = 0$
$\approx \theta'' + c\theta = 0$ (Now we have a linear equation)

Linear ODE of 2nd Order

$x'' + p(t)x' + q(t)x = f(t)$

where $p(t)$, $q(t)$, $f(t)$ are given.

Ex) $x'' + 3x' + 2x = 0$

$x'' + 5tx' + e^t x = \sin t$

When $f(t) \equiv 0$, we have a homogeneous ODE

↳ $f(t) = 0$ for all t means $\equiv$

Homogeneous ODE of 2nd order:

$x'' + p(t)x' + q(t)x = 0$

Ex) $x'' + 3x' + 2x = 0$

Try: $x_1 = t$, $x_2 = e^t$, $x_3 = e^{-t}$, $x_4 = e^{-2t}$

$x_1 = t$, $x_1' = 1$, $x_1'' = 0$

$0 + 3(1) + 2t = 0 \Rightarrow 3 + 2t \neq 0$

$x_2 = e^t$, $x_2' = e^t$, $x_2'' = e^t$

$e^t + 3e^t + 2e^t = 0 \Rightarrow 6e^t \neq 0$

$x_3 = e^{-t}$, $x_3' = -e^{-t}$, $x_3'' = e^{-t}$

$e^{-t} + 3(-e^{-t}) + 2e^{-t} = 0 \Rightarrow 0 = 0$ ✓

$x_4 = e^{-2t}$, $x_4' = -2e^{-2t}$, $x_4'' = 4e^{-2t}$

$4e^{-2t} + 3(-2e^{-2t}) + 2(e^{-2t}) = 0 \Rightarrow 0 = 0$

So x_3, x_4 are solutions!