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HW Note:

9.1 - iv) $\frac{1}{u}\int uq\,dx = \frac{1}{e^{-e^{-x}}}\int e^{-e^{-x}}\,dx$

$y = e^{e^{-x}}\left(\int_0^x e^{-e^{-t}}\,dt + c\right)$

@ $y(0) = e$

$e = e^{e^{-0}}(0 + C) \rightarrow C = 1$

Linear Dependence / independence

(Think of a function like an infinite vector)

$v = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $f(t) = t^2 \begin{bmatrix} 1 \\ 4 \\ 9 \\ \vdots \end{bmatrix}$ Bit of a strech but still...

$x_1(t)$ and $x_2(t)$ are linearly independent if

$c_1x_1(t) + c_2x_2(t) = 0$ for all t only have trivial solutions c_1, c_2.

(Only have solutions if $c_1 = c_2 = 0$)

Ex) Are t and t^2 linearly independent?

$c_1t + c_2t^2 = 0$ for all t.

Now we have infinitly many equations (for all t)

$t = 1 \Rightarrow c_1 + c_2 = 0$ > $(c_1 + c_2 = 0)2$

$t = 2 \Rightarrow 2c_1 + 4c_2 = 0$ $\quad 2c_1 + 4c_2 = 0$

$\vdots$

$$\begin{array}{r} 2c_1 + 2c_2 = 0 \\ -\ 2c_1 + 4c_2 = 0 \\ \hline 0 - 2c_2 = 0 \\ c_2 = 0 \end{array}$$

$c_1 = 0 - c_1 \rightarrow c_1 = 0 - 0 = 0$

Because $c_1 = c_2 = 0$, then sometimes t & t^2 are linearly independent.

But $t \cdot t + (-1)t^2 = 0$ So why does this not work?

t is not constant $\rightarrow$ It cannot be used as a constant scaling factor.

So we ~~cant~~ use to say t and t^2 are lin. dep.

Now with more equations:

$x_1(t), x_2(t), \ldots x_n(t)$ are lin. ind. if the equation $c_1 x_1(t) + c_2 x_2(t) + \ldots + c_n x_n(t) = 0$ for all t only has trivial solutions $c_1 = c_2 = \ldots = c_n = 0$.

Ex) $1, t, t^2, t^3, \ldots, t^{10}$. Are lin. ind.?

$c_0(1) + c_1(t) + c_2(t^3) + \ldots + c_{10}(t^{10}) = 0 \ \forall t$.

We still have infinite equations to play with but if we use 11 equations to solve for 11 unknowns Tuan will retire early.

Easier way: $t = 0: c_0 = 0$

$t(c_1 + c_2 t + c_3 t^2 + \ldots + c_{10} t^9) = 0 \ \forall t$ (So $t = 0$)

Plug in $t = 0: c_1 = 0$

Like a sniper, isolate and kill one at a time

OR

A 10^{th} degree polynomial has at most 10 roots and if one of the C's is 0 then the # roots might change but can never exceed the degree.

Because this poly has ∞ many roots, $C_0 = C_1 = C_2 = \ldots = C_{10} = 0$.

Now an example of lin dep.:

Ex) $\sin^2 t, \cos^2 t, 1$

$(1)\sin^2(t) + (1)\cos^2 t + (-1)(1) = 0$

So $c_1 = 1$, $c_2 = 1$ and $c_3 = -1$

(because $\sin^2 t + \cos^2 t = 1$)

But this is rare, linear independence is much more common

↳ Almost always.

Wronskian Function of $x_1(t), x_2(t), \dots, x_n(t)$ is

$$W(t)=\begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & \dots & x_n(t) \\ x_1'(t) & x_2'(t) & x_3'(t) & \dots & x_n'(t) \\ x_1''(t) & x_2''(t) & x_3''(t) & \dots & x_n''(t) \\ & & \vdots & & \\ x_1^{(n-1)}(t) & x_2^{(n-1)}(t) & x_3^{(n-1)}(t) & \dots & x_n^{(n-1)}(t) \end{bmatrix}$$

Should be an nxn matrix.

If $W(t) \neq 0$ for some t then $x_1(t), x_2(t), \dots x_n(t)$ are lin ind.

Why?

$c_1x_1(t) + c_2x_2(t) + \dots + c_nx_n(t) = 0 \ \forall t$

So $c_1x_1'(t) + \qquad + c_nx_n'(t) = 0 \ \forall t$

$c_1x_1''(t) + \qquad + c_nx_n''(t) = 0 \ \forall t$ for all derivatives.

So if we scale each column by a constant in $W(t)$ then we get 0.

But we know the columns of $W(t)$ are lin ind because the $\det(W(t)) \neq 0 \ \forall t$.

$$c_1\begin{bmatrix} x_1 t \\ x_1' t \\ \vdots \end{bmatrix} + \dots + c_n\begin{bmatrix} x_n(t) \\ x_n'(t) \\ \vdots \end{bmatrix} = 0$$

$\det(W(t)) = 0$ means

So the equations are lin. ind. because $c_1 \to c_n = 0$

This is a hard way to check this but it does work.

(We would need to find det of nxn matrix)

(If $\det \neq 0$ then lin ind)

But we will use this equation later...