Lecture 2

Wednesday, April 5, 2023 1:20 AM

* Questions....

A differential equation is an equation involving a function and one or more of its derivatives.

Ex $x' + x = 0$, $x'' + 2x' = t^2$, $e^x t + x' = 1$, ...

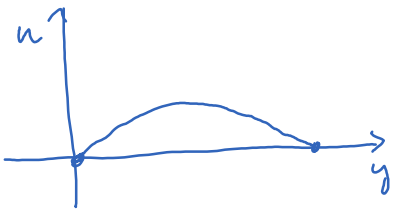
There is no general method that can solve all differential eqs. We will classify differential equations into groups and use different methods for each group.

Classification of differential equations

* Number of variables.

$x = x(t)$ ⇝ ordinary differential equation (ODE)

$u = u(x,y)$ ⇝ partial differential equation (PDE)



$u_{xx} = u_{yy}$: wave equation

$u_x = u_{yy}$

heat equation

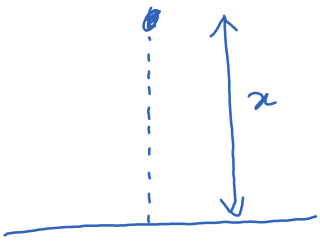
* Order of a differential equation is the highest order of derivative that appears in the equation.

Ex $x'x''' + e^x = 2t$ $\rightsquigarrow$ 3rd order ODE, or ODE of order 3

In this course, we will mainly focus on 1st and 2nd order ODE.

Most differential equations coming from physics are of this type.

Ex: falling object



$F = ma$

$a = x''$

$F = mg - \alpha v = mg - \alpha x'$ (friction)

$\rightsquigarrow$ $mg - \alpha x' = mx''$, a second order ODE

* Linear vs nonlinear:

A linear ODE of order n is of the form

$$a_n(t)\, x^{(n)} + a_{n-1}(t)\, x^{(n-1)} + \cdots + a_1(t)\, x' + a_0(t)\, x = f(t)$$

Ex $t x''' + \sin(t)\, x' + e^t x = 16 \cos t$ $\rightsquigarrow$ linear ODE of order 3

$x'' x + x' = 0$ $\rightsquigarrow$ nonlinear ODE

Types of solutions

(1) Fully explicit: $x = e^t \cos t$ for example,

(2) Less explicit: $x = \int_0^t e^{-s^2} ds$, $x = \sum_{n=1}^{\infty} \frac{t^n}{n^2}$ for example

(3) Implicit: $\sin(x) + x\cos(t) = 5$

Our next goal is to solve the autonomous ODE of first order.

This kind of ODE is of the form

$$\underbrace{x' = f(x)}$$

no direct dependence on t

All physical laws are autonomous: they do not change in time.

For example: $F = ma$ (no explicit dependance on time)

Note: In the textbook, they write $\dot{x}$ and $\ddot{x}$ instead of x' and x'' when the variable is time.