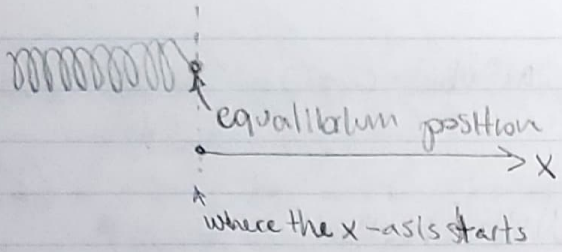
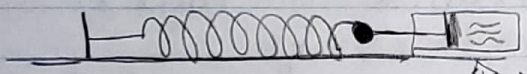


$\underbrace{mx''}_{\text{Net force (No friction)}} = \underbrace{-kx}_{\text{Spring force}}$

$x'' + \frac{k}{m}x = 0$

$\omega = \sqrt{k/m}$

$x = c_1\cos(\omega t) + c_2\sin(\omega t)$



$\underbrace{mx''}_{\text{Net force}} = \underbrace{-kx}_{\text{spring force}} - \underbrace{\mu x'}_{\text{Friction}}$ (μ is a positive constant)

$mx'' + \mu x' + kx = 0$

Char eqn: $mr^2 + \mu r + k = 0$

- 2 distinct roots
- 1 double root
- 2 complex roots

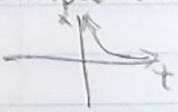
(b^2-4ac) $\Delta = \mu^2 - 4mk$ $\Rightarrow$ If Δ is pos, $\mu > \sqrt{4mk}$ $\Rightarrow$ 2 distinct roots

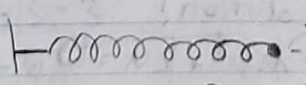
$\Delta = 0$ then $\mu = \sqrt{4mk}$ $\Rightarrow$ double root

Δ is neg then $\mu < \sqrt{4mk}$ $\Rightarrow$ 2 complex roots

- $\mu > \sqrt{4mk}$: roots r_1, r_2 are real and negative

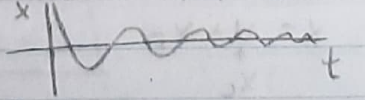
  solution $c_1e^{r_1t} + c_2te^{r_2t} = x$ ← Decay function





- $\mu < \sqrt{4mk}$: roots $r = \alpha \pm i\beta$ Sol: $x = e^{\alpha t}(c_1\cos\beta t + c_2\sin\beta t)$

  $r = \frac{-\mu \pm \sqrt{\Delta}}{2m}$ $x = -\frac{\mu}{2m} < 0$



- $\mu = \sqrt{4mk}$: roots double, $x = (c_1t + c_2)e^{rt}$ $r < 0$

  exponentially fast