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Recall: If $ax'' + bx' + cx = f(t)$

If $f(t) = P(t)e^{at}$ with $P(x)$ a poly of degree n

Then we guess $x_p = \underbrace{Q(t)}_{\text{General w/ degree of poly}} e^{at}$

If x_p contains any parts of x_c then the guess fails. Then we increase the degree by 1 and guess again.

Ex) $x'' - 2x' + x = te^t$

$x_{g_1} = (At+B)e^t = Ate^t + Be^t$

$x_c \Rightarrow x'' - 2x' + x = 0$

$r^2 - 2r + 1 = 0 \Rightarrow (r-1)(r-1) \quad r = 1$

$x_1 = e^t \quad x_2 = te^t$

$x_c = c_1 e^t + c_2 te^t$

Has everything in common with x_c, so can't guess this

$x_{g_2} = (At^2 + Bt + C)e^t = At^2e^t + \underbrace{Bte^t + Ce^t}$

This is in x_c too! So B and C are of no help because they (are redundent) are accounted for in x_c.

So the part we care about is $At^2e^t \leftarrow$ New guess!

But we need 2 unknowns! So raise it again!

$x_{g_3} = (At^3 + Bt^2 + Ct + D)e^t = \underbrace{At^3e^t + Bt^2e^t}_{\text{Give 2 unknowns}} + \underbrace{Cte^t + D}_{\text{These are still unessasary}}$

So our guess is $At^3e^t + Bt^2e^t = x_p = t^2(Ate^t + Be^t)$

$x = At^3e^t + Bt^2e^t$, $x' = 3At^2e^t + At^3e^t + 2Bte^t + Bt^2e^t$

$x' = (3At^2 + At^3 + 2Bt + Bt^2)e^t$

$x'' = (6At + 3At^2 + 2B + 2Bt)e^t + (3At^2 + At^3 + 2Bt + Bt^2)e^t$

$= (At^3 + (6A+B)t^2 + (4B+6A)t + 2B)e^t$

$$x'' = (At^3 + (6A+B)t^2 + (6A+4B)t + 2B)e^t$$
$$-2x' = (-2At^3 + (-6A-2B)t^2 + -4Bt\quad)e^t$$
$$x = (At^3 + Bt^2\quad)e^t$$
$$0t^3 + 0t^2 \quad 6At + 2B$$

$te^t = (6At + 2B)e^t$

$te^t = 6Ate^t \qquad 0 = 2B$

$t = 6At \qquad 0 = B$

$1 = 6A$

$\frac{1}{6} = A$

So $\boxed{x_p = \frac{1}{6}t^3e^t}$

General Solution: $x = x_c + x_p$

$x = c_1e^t + c_2te^t + \frac{1}{6}t^3e^t$

In the case $f(t) = P(t)\{trig\}$

$f(t) = P(t)\cdot\cos(t)$ OR $f(t) = P(t)\cdot\sin(t)$ with $p(t)$ a poly

Then guess $x_p = Q(t)(A\cos(at) + B\sin(at))$

Ex) $x'' - 2x' + x = \cos(2t)$

guess $x_p = A\cos(2t) + B\sin(2t)$

$x_c = c_1e^t + c_2te^t$

↳ Nothing's in x_c so this is the guess we want.

Ex) $x'' + 4x = \cos(2t)$

guess $x_p = A\cos(2t) + B\sin(2t)$

$x_c \Rightarrow r \pm 2i \quad \alpha = 0,\ \beta = 2$ so $x_c = e^{\alpha t}(c_1\cos\beta t + c_2\sin\beta t)$

$= c_1\cos(2t) + c_2\sin(2t)$

↳ This is same as our guess, so the guess won't work!

Raise the power of Q!

Right now $\deg(Q(t)) = 0$

guess 2: $x_p = (Ct + D)(A\cos(2t) + B\sin(2t))$

$= Ct(A\cos(2t) + B\sin(2t)) + \underbrace{D(A\cos(2t) + B\sin(2t))}_{\in x_c}$

Better guess: $x_p = A_1t\cos(2t) + B_1t\sin(2t)$

What if the right side is mixed?

Ex) $x'' - 2x' + x = e^t + \cos(2t)$

1) Find an x_{p_1} for $x'' - 2x' + x = e^t$ (x_1)
2) Find an x_{p_2} for $x'' - 2x' + x = \cos(2t)$ (x_2)
3) $x_p = x_1 + x_2$

Superposition principle
(Can do this because this is linear)

5/18 We can now solve $ax'' + bx' + cx = f(t)$ with special types of $f(t)$ by guessing a particular sol.

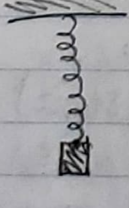
Ex) Polynomial, e^{2t}, $\cos(3t)$, ...

But what if $f(t) = \frac{e^t}{t}$

The method we are doing now is called the <u>Method of Undetermined coefficents</u>

But for something like $f(t) = \frac{e^t}{t}$ we will use the <u>Method of variation of parameters</u>. (We will talk about this tomorrow)

<u>Applications</u> (of nonhomogeneous 2nd order ODE's)



$\underset{\text{gravity}}{mx''} + \underset{\text{Friction}}{\mu x'} - \underset{\text{Spring force}}{kx} = F(t)$

If you don't interfere $f = 0$
But if you do then f is some external force.
(Ex. You hold the spring and move up/down on a beat)