What if the right side is mixed?

Ex) $x'' - 2x' + x = e^t + \cos(2t)$

1) Find an x_{p_1} for $x'' - 2x' + x = e^t$ (x_1)
2) Find an x_{p_2} for $x'' - 2x' + x = \cos(2t)$ (x_2)
3) $x_p = x_1 + x_2$

Super position principle
(Can do this because this is linear)

5/18 We can now solve $ax'' + bx' + cx = f(t)$ with special types of $f(t)$ by guessing a particular sol.

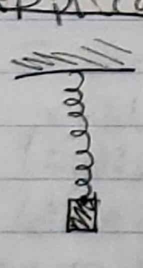
Ex) Polynomial, e^{2t}, $\cos(3t)$, ...

But what if $f(t) = \frac{e^t}{t}$

The method we are doing now is called the <u>Method of Undetermined coefficents</u>

But for something like $f(t) = \frac{e^t}{t}$ we will use the <u>Method of variation of parameters.</u> (We will talk about this tomorrow)

<u>Applications</u> (of nonhomogeneous 2^{nd} order ODE's)



$$\underset{\text{gravity}}{m}x'' + \underset{\text{Friction}}{\mu}x' - \underset{\substack{\text{Spring}\\ \text{force}}}{k}x = F(t)$$

If you don't interfice $f = 0$
But if you do then f is some external force.
(Ex. You hold the spring and move up/down on a beat)

If $F(t)$ is a periodic forcing, is the vibration also periodic with the same frequency?

First, consider the case of no friction: $\mu = 0$

$mx'' + kx = f(t)$ ← W/Force

W/out force: $mx'' + kx = 0$

There are two frequencies going on...

Frequency of the force: ω ← We control this

Frequency of the system: ω_0 ← Physics controls this

Reminder: $mx'' + kx = 0 \Rightarrow x'' + \frac{k}{m}x = 0$

$\omega_0 = \sqrt{\frac{k}{m}}$

$c_1 \cos \omega_0 t + c_2 \sin \omega_0 t$

Consider $f(t) = a \cos \omega t$

Amplitude (How much up/down you move the spring)

ODE: $mx'' + k(x) = a\cos(\omega t)$

Assume: $m = 1$, $k = 4$, $\omega = 3$, $a = 2$

☆ $x'' + 4x = 2\cos(3t)$

$x_c = c_1 \cos \omega_0 t + c_2 \sin \omega_0 t$ $\quad \omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{4}{1}} = 2$

$x_c = c_1 \cos 2t + c_2 \sin 2t$

$x_p = A\cos(3t) + B\sin(3t)$

$x' = -3A\sin(3t) + 3B\cos(3t)$

$x'' = -9A\cos(3t) - 9B\sin(3t)$

$-9A\cos(3t) - 9B\sin(3t) + 4(A\cos(3t) + B\sin(3t)) = 2\cos(3t)$

$(-9A + 4A)\cos(3t) + (-9B + 4B)\sin(3t)) = 2\cos(3t)$

$-5A\cos(3t) = 2\cos(3t)$ $\quad -5B\sin(3t) = 0\sin(3t)$

$-5A = 2$ $\quad -5B = 0$

$A = -\frac{2}{5}$ $\quad B = 0$

$x_p = -\frac{2}{5}\cos(3t)$

$x = \underbrace{c_1\cos(2t) + c_2\sin(2t)}_{\text{Period} = \pi} - \underbrace{\frac{2}{5}\cos(3t)}_{\text{Period} = \frac{2\pi}{3}}$

after 2π the whole system repeats so the whole system is periodic w/ period $= 2\pi$

What happens if the natural frequency is equal to the forcing frequency?

Ex) $\overset{a}{x''}+\overset{m}{4}x=\overset{k}{2}\cos(\overset{w}{2}t)$, $m=1$ $k=4$ $w=2$ $a=2$

$x_c = c_1\cos w_0 t + c_2 \sin w_0 t$

$x_c = c_1 \cos 2t + c_2 \sin 2t$

$x_p = At\cos(2t) + Bt\sin(2t)$

$x_p' = A\cos(2t) + 2At\cos(2t) + B\sin(2t) + 2Bt\sin(2t)$

$x_p'' = 2A\cos(2t) + 2A\cos(2t) + 4At\cos(2t) + 2B\sin(2t) + 2B\sin(2t) + 4Bt\sin(2t)$

$= 4A\cos(2t) + 4At\cos(2t) + 4B\sin(2t) + 4Bt\sin(2t)$

5/19 $x'' + 4x = \cos(2t)$

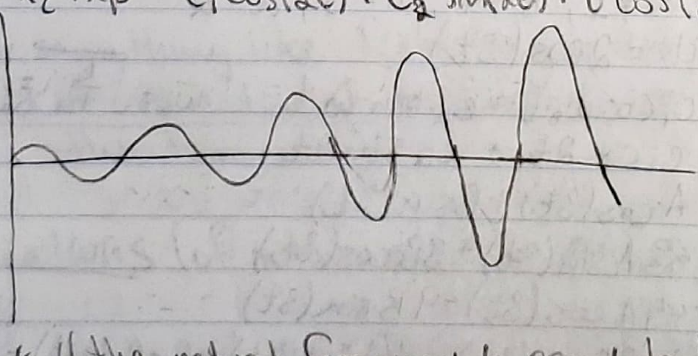
$x_p = At\cos(2t) + Bt\sin(2t)$

Lets assume $A=1, B=0$ (lots of work)

$x_p = t\cos(2t)$

General Sol.

$x = x_c + x_p = c_1\cos(2t) + c_2\sin(2t) + t\cos(2t)$



This is if the natural frequency is equal to the forcing frequency.

What happens if we add friction?