Consider $F(t) = a\cos(\omega t)$ (different ω gives different frequencies)
The solution vibrates the most (largest amplitude)
When $\omega = \sqrt{\frac{k}{m}} \cdot \sqrt{1 - \frac{c^2}{2mk}}$ (Resonance)
This is what happens in real life, because there is always friction.
(Think about a swing & pushing someone)

5/22 Solve equations of the form $ax'' + bx' + cx = f(t)$
- Find x_c & x_p and add
  - Find x_p through the method of undetermined coefficents. (for certian types of $f(t)$)
    - learn this first to learn what the heck we doing

Alternatively, we can find a particular solution using the Method of Variation of Parameters

Suppose $x_c = c_1x_1 + c_2x_2$
← Complementary Solution

If I plug this into $ax'' + bx' + cx = f(t)$
we get $ax_c'' + bx_c' + cx_c = 0 \;\; \forall c_1, c_2$, c_1, c_2 are constants

Idea: Allow c_1, c_2 to be functions of t (rather than constants)
We'll find c_1, c_2 so that the right hand side is equal to f.
For ease, lets rename C_1 as u_1 and C_2 as u_2
- u_1, u_2 are functions of t (not constants)

We are seeking for a particular solution of the form
$x = u_1x_1 + u_2x_2$, where x_1, x_2 are linearly independent solutions of the homogeneous equation.
We need to determine u_1, u_2

$ax'' + bx' + cx = f(t)$

$x = u_1 x_1 + u_2 x_2$

$x' = u_1' x_1 + u_1 x_1' + u_2' x_2 + u_2 x_2'$

$x'' = u_1'' x_1 + u_1' x_1' + u_1' x_1' + u_1 x_1'' + u_2'' x_2 + u_2' x_2' + u_2' x_2' + u_2 x_2''$

$= u_1'' x_1 + 2u_1' x_1' + u_1 x_1'' + u_2'' x_2 + 2u_2' x_2' + u_2 x_2''$

Keep in mind $ax_1'' + bx_1' + cx_1 = 0$
$ax_2'' + bx_2' + cx_2 = 0$

You killed 6 of 12! That's a great mortality Rate! -Tuan

$+ a(u_1'' x_1 + u_2'' x_2 + \underline{u_1 x_1''} + u_2 x_2'' + 2u_1' x_1' + 2u_2' x_2')$
$+ b(u_1' x_1 + \underline{u_1 x_1'} + u_2' x_2 + u_2 x_2')$
$c(\underline{u_1 x_1} + u_2 x_2)$

$u_1(\underbrace{ax_1'' + bx_1' + cx_1}_{0}) + u_2(\underbrace{ax_2'' + bx_2' + cx_2}_{0}) = f(t)$

$\Rightarrow f(t) = au_1'' x_1 + au_2'' x_2 + 2au_1' x_1' + 2au_2' x_2' + bu_1' x_1 + bu_2' x_2$

Now we need u_1 & u_2 (1 eq, but 2 unknowns)
↳ More freedom to pick & choose

We can set one unknown to 0. This will give us 2 eq w/ 2 unknowns

If we let $(u_1' x_1 + u_2' x_2) = 0$ then how can we relate a different term to it?

$f = a(u_1'' x_1 + u_2'' x_2) + 2a(u_1' x_1' + u_2' x_2') + b(u_1' x_1 + u_2' x_2)$

$0 = (u_1' x_1 + u_2' x_2)' = u_1'' x_1 + u_1' x_1' + u_2' x_2' + u_2'' x_2$

So $0 = \underbrace{u_1'' x_1 + u_2'' x_2}_{-y} + \underbrace{u_1' x_1' + u_2' x_2'}_{y}$

$f = a(\underbrace{u_1'' x_1 + u_2'' x_2}_{-y}) + 2a(\underbrace{u_1' x_1' + u_2' x_2'}_{y}) = f = a(-y) + 2ay = 2a$

$y = \frac{-f}{a} \Rightarrow u_1' x_1' + u_2' x_2' = \frac{f}{a}$ So now we have 2 equations!

$\begin{cases} u_1' x_1 + u_2' x_2 = 0 \\ u_1' x_1' + u_2' x_2' = \frac{f}{a} \end{cases}$ $\Rightarrow x_1$ & x_2 are known, so is $\frac{f}{a}$
u_1 & u_2 are our unknowns.

$(u_1' x_1 + u_2' x_2 = 0)\, x_1' \Rightarrow$
$-\,(u_1' x_1' + u_2' x_2' = \frac{f}{a})\, x_1$

$(x_1' x_2 - x_1 x_2')\, u_2' = -x_1\left(\frac{f}{a}\right) = x_1(g)$

$u_2' = \dfrac{-x_1\left(\frac{f}{a}\right)}{x_1' x_2 - x_1 x_2'}$ $\quad u_1' = \dfrac{x_2\left(\frac{f}{a}\right)}{x_1' x_2 - x_1 x_2'}$ (Check it)

Therefore, $u_1' = \dfrac{-x_2 g}{W}$

$u_2' = \dfrac{x_1 g}{W}$ $\Rightarrow W$ is wronskian eq.