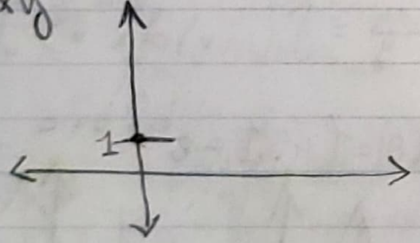
6/1

Eulers Method: A numerical method to solve an ODE
We will use it for initial value problems of
$y' = f(x,y)$, $y(x_0) = y_0$ (Note: Like power series method, no specific form of ODE is required for the method to apply.)

Ex) $y' = xy$ ← Can be solved using seperatable equations
$y' = x + y$ ← Can be solved using integrating factors
$y' = y^x$
$y' = \sin(xy)$ } These we dont have a good method to use yet

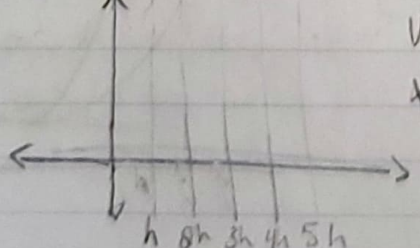
Consider $y' = xy$, want to solve knowing the initial condition, $y(0) = 1$.
In many cases, the solution is $y = y(x)$
But thats not the case for power series: $y = a_0 + a_1x^1 + a_2x^2 + \dots$
So the solutions are the same(ish) but are described differently.
Another way to think about a solution is a graph of $y =$ _____
Eulers method looks for a solution as a graph of $y = y(x)$

$y' = xy$



((0,1) is the initial condition)
$y'(x) = x\,y(x)$ @ $x = 0$
$y'(0) = 0\,y(0)$
$y'(0) = 0$ ← Slope @ $x = 0$

Find points by



We discretize the x-axis with quadrants
$x_0 = 0$
$x_1 = h$
$x_2 = 2h$
$x_3 = 3h$
$x_4 = 4h$
} H is a stepsize, if h is big it has a hard time looking far away

But what is $y(x_k)$?
Sub $x = x_k$ in ⇒ $y'(x_k) = x_k\,y\,x_k$

$\therefore \quad y'(x_k) = x_k\, y(x_k)$

$y'(x_k) = \lim_{\delta \to 0} \dfrac{y(x_k+\delta) - y(x_k)}{\delta}$

Note: H is small by defn. So we can use $\delta = h$ as an approximation.

$\delta = h \Rightarrow \dfrac{y'(x_{k+1}) - y(x_k)}{h}$

The ODE is approx. an $\dfrac{y(x_{n+1}) - y(x_k)}{h} \approx x_k y(x_k)$

And we know by power series the first coeff is 1.

Plus! This another recursion method.

Now, mult both sides by h.

$\dfrac{y(x_{n+1}) - y(k)}{h}$

$y(x_{k+1}) - y(x_k) = h x_k y(x_k)$

$\Rightarrow y(x_{k+1}) \approx (1 + h x_k)\, y(x_k)$

Now we can start approximation.

$y(x_0) = y_0 = 1$ h is your choice, unless given. But H should be small w/ relation to x range.

0 .2 .4 .6 ...

$x_0 \quad x_1 \quad x_2 \quad x_3$

Choose $h = 0.2$, approximate $y(0), y(0.2), y(0.4), y(0.8), y(1)$

$y(0) = 1$

$y(0.2) \approx y(x_1) \approx (1 + h x_0)\, y(x_0)$

$\approx (1 + (0.2)(0))(1) = 1$

$y(0.4) \approx y(x_2) \approx (1 + h x_1)\, y(x_1)$

$\approx (1 + (.2)(.2))1 = 1.04$

$y(0.6) \approx y(x_3) \approx (1 + h x_2)\, y(x_2)$

$(1 + (.2)(.4))(1.04) = 1.1232$