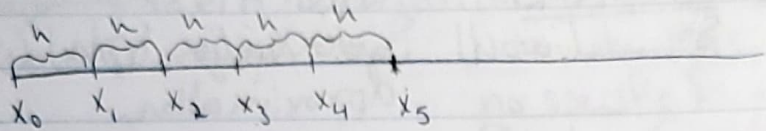
6/2

$y' = f(x,y)\,,\; y(x_0) = y_0$



$x_0 = x_0$

$x_1 = x_0 + h$

$x_2 = x_0 + 2h$ $\qquad\qquad y(x_n) \approx y_n$

$x_n = x_0 + nh$

$$y'(x_n) \approx \frac{y(x_{n+1}) - y(x_n)}{x_{n+1} - x_n} = \frac{y_{n+1} - y_n}{h}$$

$$y_{n+1} - y_n = h\, f(x_n, y_n)$$

Recursion formula $\rightarrow$ $y_{n+1} = y_n + h f(x_n, y_n)$

The smaller the h, the better the approximation

$f(x_n, y_n) = x_n + y_n$

$y' = x + y\,,\; y(0) = 1$ => Stuff done on Octave (Free MatLab) (octave-online.net)

Sign in (Blue button), email verification, change password

Save before running

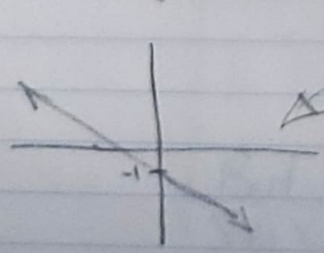
- Make an empty file (Plus sign top left)
- Lets solve the ODE on the interval $x \in [0,10]$
- Choose $h = 0.5$
- We want $x_0 = 0$, $x_1 = h$, $x_2 = 2h$, ... but Matlab starts at 1, Octave starts at 0.
- In output area put x(1), then x(2) (to check), x(20), x(21) (to check)
- Can also do length(x)
- Make $y_0 = -1$

Script

```
h = 0.5;
x = [0:h:10];
y(1) = -1;
N = length(x);
for n = 1:(N-1)
    y(n+1) = y(n)+h*(xn + yn);
end
plot(x,y)
```

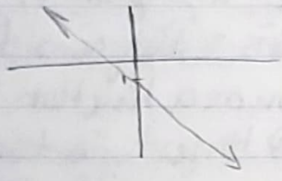
Change to 1 to get different graph

Makes x & y same length



Solve $y' = x + y$ on Jupyter-Lab

```
DSolve[{y'[x] == x + y[x], y[0] == -1}, y[x], x]
    y[x] = -1 - x
```



But

```
DSolve[{y'[x] == x + y[x], y[0] == 1}, y[x], x]
    y[x] = -1 + 2e^x - x
```

MatLab/Octave:

```
h = 0.5;
x = [0:h:10];
y(1) = 1;
N = length(x);
for n = 1:(N-1)
    y(n+1) = y(n) + h*(x_n + y_n);
end
for n = 1:(N-1)
    yex(x) = -1 - 2 exp(x(n)) - x(n);
end
plot(x, y)
hold on
plot(x, yex)
```

Approx Solution → (first loop)

Actual Solution → (second loop)

To plot them together: plot(x,y) / hold on / plot(x,yex)

Then try lowering h.

$y_{ex}(x) = -1 + 2e^{x} - x$

$y_{ex}(x_n) = -1 + 2e^{x_n} - x_n$