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$$\begin{cases} x_1' = 2x_1 + x_2 \\ x_2' = x_1 + 2x_2 \end{cases}$$

Last time: $x_2 = x_1' - 2x_1$

$\Rightarrow x_1'' - 2x_1' = x_1 + 2x_1' - 4x_1$

$\Rightarrow 0 = x_1'' - 4x_1' + 3x_1 \leftarrow$ Solve

Use $x_1(0) = 1$ & $x_2(0) = 1$ to find $x_1'(0)$

Find c_1 & c_2

use x_1 to find x_2.

In Linear Algebra:

$$\begin{cases} 2x + 3y = 5 \\ x - y = 6 \end{cases} \Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$\left[\begin{array}{cc:c} 2 & 3 & 5 \\ 1 & -1 & 6 \end{array}\right] \text{ \& Solve}$$

We will find a way to solve a problem like yesterdays by a matrix method.

$$\begin{cases} x_1' = 2x_1 + x_2 \\ x_2' = x_1 + 2x_2 \end{cases} \Rightarrow \underbrace{\begin{bmatrix} x_1' \\ x_2' \end{bmatrix}}_{X'} = \underbrace{\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}_{\substack{\text{Matrix of} \\ \text{Coeff.} \\ A}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{\substack{\text{Column} \\ \text{of Unknowns} \\ X}}$$

$X' = AX \Leftarrow$ We will solve this

$X' = AX$ How to solve?

Observations:

1) If X_1 & X_2 are solutions, then $X_1 + X_2$ is also a solution.

$X_1' = AX_1$ and $X_2' = AX_2$

$X_1' + X_2' = A(X_1 + X_2)$

2) If X is a solution and c is a constant number, then cX is also a solution.

$(cX)' = cX' = cAX = A(cX)$

Thm: Let A be a constant $n \times n$ matrix (no variables in A). If $X_1, X_2, \ldots, X_n$ are lin ind. solutions to the equation $X' = AX$, then a general solution to this equation is of the form $X = c_1X_1 + c_2X_2 + \ldots + c_nX_n$. (Remember that X_n are vectors)

Linear independence: $X = \begin{bmatrix} t \\ t^2 \end{bmatrix}$ $Y = \begin{bmatrix} 1 \\ t+1 \end{bmatrix}$

Consider $c_1X + c_2Y = 0$

$$c_1\begin{bmatrix} t \\ t^2 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ t+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$ lin. ind if $c_1, c_2 = 0$

$\begin{cases} c_1t + c_2 = 0 \\ c_1t^2 + c_2(t+1) = 0 \end{cases}$ Pick $t = 0$, $c_2 = 0$ so $c_1 = 0$.

So the vectors X & Y are lin. ind.

★ Remind how to find det.

Recall: $u = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $v = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$ $w = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ Check if u, v, w are lin ind. If $\det(uvw) \neq 0$ then u, v, w lin ind.

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & -1 & 1 \\ 3 & 1 & 3 \end{bmatrix} \xrightarrow{R_2 - R_1} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 3 & 1 & 3 \end{bmatrix} \xrightarrow{R_3 + R_1} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 4 & 0 & 3 \end{bmatrix} \ldots\ldots$$

If the vector depends on t then $\det(uvw) \neq 0$ still means lin. ind but otherwise does not mean lin dep.

Ex of det(uvw) = 0 depending on t does not always mean that uvw are lin dep.

$u = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $v = \begin{bmatrix} t \\ t \end{bmatrix}$ lin dep because v is a scalar of u.

$c_1 u + c_2 v = 0$, c_1 & c_2 are constant

$c_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} t \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$c_1 + c_2 t = 0 \ \forall t$

$\Rightarrow c_1, c_2 = 0$ so v and u are lin ind!

Conflicting

right

Why Conflicting?

$v = tu$ but t is not a constant!

So if det ≠ 0 then lin ind.

if det = 0 then needs double checked to see for sure.

$$\underset{X'}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}'} = \underset{A}{\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}} \underset{X}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}$$

From last time ↓

$$\begin{cases} x_1 = c_1 e^{t} + c_2 e^{3t} \\ x_2 = -c_1 e^{t} + 3c_2 e^{3t} \end{cases}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c_1 e^{t} + c_2 e^{3t} \\ -c_1 e^{t} + 3c_2 e^{3t} \end{bmatrix}$$

$$= \begin{bmatrix} c_1 e^{t} \\ -c_1 e^{t} \end{bmatrix} + \begin{bmatrix} c_2 e^{3t} \\ 3c_2 e^{3t} \end{bmatrix}$$

$$= c_1 \underbrace{e^{t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}}_{X_1} + c_2 \underbrace{e^{3t} \begin{bmatrix} 1 \\ 3 \end{bmatrix}}_{X_2}$$

To find the special solutions $X_1, X_2, \ldots, X_n$, we seek them in the form $X = e^{st} \underset{\uparrow}{V}$

constant vector.