Lecture 4

Friday, April 7, 2023 10:43 AM

* Questions ...

Autonomous ODE $x' = f(x)$ is difficult to solve even with relatively simple function f. In such a case, we resort to a qualitative approach rather than quantitative approach. We ask questions such that

- Is the solution $x(t)$ increasing or decreasing or not monotonic ?
- Does the solution have a limit as $t \to \infty$ or $t \to -\infty$?
- Is the solution $x(t)$ bounded ?
- Is the solution periodic ?
- What the graph of the solution look like?
- How does the solution depend on the initial condition $x(0)$?

<u>Ex</u> $\quad x' = \underbrace{x^2 + x}_{f(x)}$

The values x such that $f(x) = 0$ is called the <u>stationary points</u> of the ODE. In some textbooks, these are called the <u>equilibrium states</u>.

In this problem, 0 and -1 are the stationary points.

If the initial state $x(0) = x_0$ is equal to one of the equilibrium states then it remains there for all time.

If $x_0 \in (-1, 0)$ then the ODE drives the states toward the equilibrium 0.

-1 is a stable equilibrium.

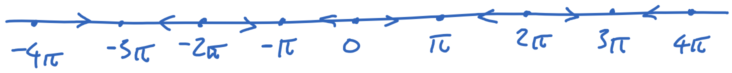
0 is an unstable equilibrium

If f is a differentiable function, a stable eq. state is an eq state where $f' < 0$. An unstable eq. state is where $f' > 0$.

Ex

$$x' = \sin x$$

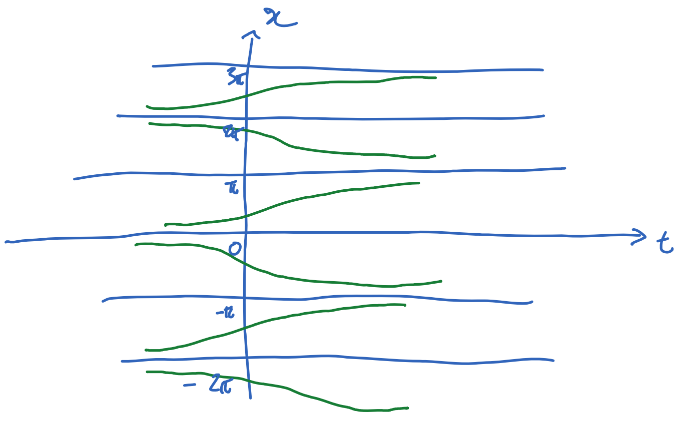
Equilibrium states are $k\pi$, $k \in \mathbb{Z}$



phase diagram, or phase portrait

The stable equilibrium states are $k\pi$, k is odd.

The unstable eq. states are $k\pi$, k is even.



behavior of solution as a function of time, depending on the initial state x_0

Ex $\quad x' = x(x-1)^2(x-2)$

Eq states are $0, 1, 2$

Sign chart for $f(x) = x(x-1)^2(x-2)$.

x		0		1		2	
x	$-$	0	$+$		$+$		$+$
$(x-1)^2$	$+$		$+$	0	$+$		$+$
$x-2$	$-$		$-$		$-$	0	$+$
$f(x)$	$+$	0	$-$	0	$-$	0	$+$