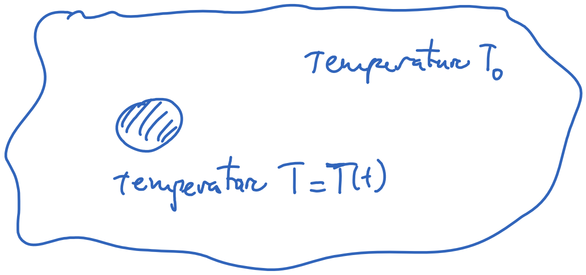
Lecture 9

Monday, April 17, 2023 8:32 AM

* Questions

Applications of linear 1st order ODEs:

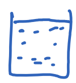
- Newton's law of cooling



$T'(t) \sim T_0 - T(t)$

$T' = \alpha(T_0 - T)$

Ex



hot water

Initial temperature = 200°F

Room temperature = 70°F

In 1 minute : 190°F

When will the glass of water be at 150°F ?

A naive approach would be to say that the temperature drops 10°F every 1 minute. That way, it would take 5 minutes to reach 150°F.

Ex Estimate the time of death.

Body temperature (alive) = 98F

Room temperature = 68°F

8AM 82°F

9AM 78°F

What is the estimated time of death?

$T' = \alpha(68 - T) \rightsquigarrow T = 68 + 30e^{-\alpha t}$

The time of death is a.

At $t = 8-a$ $\quad 82 = 68 + 30e^{-\alpha(8-a)}$

$\rightsquigarrow -\alpha(8-a) = \ln\left(\frac{14}{30}\right)$

At $t = 9-a$ $\quad 78 = 68 + 30e^{-\alpha(9-a)}$

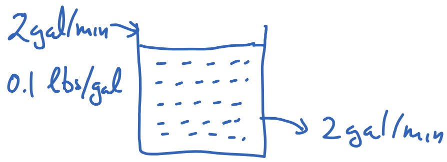
$\rightsquigarrow -\alpha(9-a) = \ln\left(\frac{10}{30}\right)$

$$\frac{8-a}{9-a} = \frac{\ln(14/30)}{\ln(10/30)} \approx \dots$$

The solve for a from here

For more history of the use of ODE in guessing the time of death, read the following article: https://maa.tandfonline.com/doi/full/10.1080/0020739X.2011.592613

Ex Mixing problem



$V(0) = 500$ gal, $x(0) = 0.01$

$x = x(t)$. salt concentration at time t.

$y = V(t)x$ = amount of salt

$y' =$ rate in $-$ rate out $= 2 \times 0.1 - x \times 2 = 0.2 - 2x$

$\rightsquigarrow V'x + Vx' = 0.2 - 2x$

$\rightsquigarrow Vx' = 0.2 - (2 + V')x$

$$\rightsquigarrow \boxed{x' + \frac{2+V'}{V}x = \frac{0.2}{V}}$$

In our problem, $V(t) = 500$ for all t. Then $V' = 0$.

Thus,
$$x' + \frac{2}{500}x = \frac{0.2}{500}, \quad x(0) = 0.01$$