Midterm: Some problems for review

The exam will be held at the Testing Center (Zabel Hall 112) from Monday 5/8 to Wednesday 5/10. The material covered is Section 1, 3, 6, 7, 8, 9, 11, 12 (not including complex roots) of the textbook. It is a closed book exam. A 4" x 6" handwritten single-sided note card is allowed. A scientific calculator is allowed (*and you will need it!*) Graphing/ programmable/ transmittable calculators are not allowed.

You should review the homework problems, the examples given in the textbook and in the lectures. It is always a good idea to study for the exam with someone. The types of problems you may be asked on the exam include:

- Classify a differential equation: order, ODE vs PDE, linear vs nonlinear, autonomous vs non-autonomous.
- For first order autonomous ODE, draw a phase diagram and behavior of solutions with different initial conditions.
- Solve an ODE using the separation of variables method or integrating factor method.
- Use the characteristic equation to solve a second order linear ODE with constant coefficients.
- Use the theorem of existence and uniqueness of solutions.
- Solve problems of population growth, Newton's law of cooling, carbon dating.

Additional problems to practice:

1) The initial value problem $y' = x\sqrt{y}$, $y(0) = 0$ has two solutions $y_1 = 0$ and $y_2 = \frac{x^4}{16}$. Explain why how the criterion for existence and uniqueness fails.

2) Consider the autonomous ODE $x' = x^3 - x$. Determine all equilibrium states. Draw the phase diagram and the diagram behavior of solutions with different initial conditions. Classify the equilibrium states into stable, unstable, semistable. If $x(1) = 1/2$, what is $\lim_{t\to\infty} x(t)$?

3) Find the general solution of $y' = 3x^2e^{-y}$ and the particular solution that satisfies the condition $y(0) = 1$.

4) Find the general solution of $y' + \frac{3y}{x+1} = (x+1)^4$.

5) While examining an wooden artifact, scientists found that the ratio of ${}^{14}C$ to ${}^{12}C$ is only 95% of the usual ratio in a living plant. According to the carbon dating method given by the ODE $N' = -kN$, approximately how long ago was the artifact made?