Problem 13.5 on page 127:

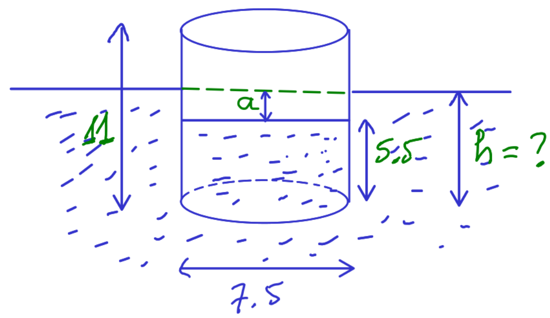

Figure 1: The can at rest

When the can is at the equilibrium position, denote by a the gap between the water level inside the can and water level outside of the can (see the picture). The water inside the can can be viewed as an extension of the water outside of the can. The occupied volume is

$$V = base\ area\ \times a = \pi r^2 a$$

At the equilibrium position, the gravity is balanced with the bouyancy force:

$$mg = F_b = V\rho g \tag{1}$$

Here, m is the weight of the can itself, not including the water inside it because this water is treated as an extension of the water outside. Thus, $m = 50$. Note that the mass density of water is $\rho = 1$ g/cm^3. Equation (1) implies

$$m = V = \pi r^2 a$$

and therefore

$$a = \frac{m}{\pi r^2} = \frac{50}{\pi (3.75)^2} \approx 1.1318$$

The can is immersed $h = a + 5.5 \approx 6.6318$ cm in the water.

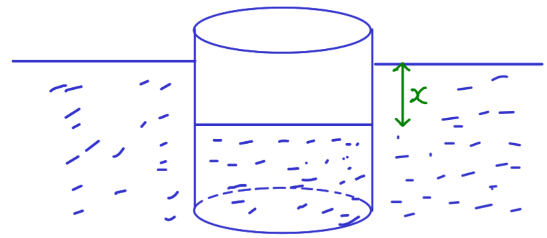

Figure 2: The can oscillates

Now consider the can in oscillation. Let $x = x(t)$ be the gap between the water level inside the can and the water level outside the can. Two forces of opposite directions applying on the can are the gravity mg and the bouyancy force $V\rho g$. By Newton's Second Law,

$$mx'' = mg - V\rho g \tag{2}$$

The occupied volume is

$$V = base\ area\ \times x = \pi r^2 x$$

Thus, equation (2) can be written as

$$x'' + \frac{\pi r^2 g}{m} x = g \tag{3}$$

Denote $\overline{x} = x - \frac{m}{\pi r^2}$. Then we can rewrite equation (3) as

$$\overline{x}'' + \frac{\pi r^2 g}{m} \overline{x} = 0$$

Let $\omega = \frac{\pi r^2 g}{m}$. Then we get $\overline{x}'' + \omega^2 x = 0$. This is a periodic oscillation with period (in second)

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{\pi r^2 g}} = 2\pi \sqrt{\frac{50}{\pi (3.75)^2 980}} \approx 0.2135$$