Final Exam: Some problems for review

You will take your Final exam at the regular classroom SCB 304 on Friday, June 28, from 1 PM to 3:50 PM. You will do it on paper (not on a computer). The material covered is Section 8.3, 8.4, 11.1-11.4. There will be some questions for extra credit. They can help you earn up to 5% of the total course score. The extra credit problems are about the topics you learned from the beginning of the course up to Midterm II. A good way to prepare for them is to review and practice Midterm I and II on WebAssign.

It is a closed book exam. No notes are allowed. You are allowed to use a scientific or graphic calculator. The trigonometric identities will be provided on the exam, so you don't have to memorize them. Phones are not allowed. You should review the homework problems, quizzes, and examples given in the lectures. It is always a good idea to study for the exam with someone. Some additional problems to practice:

1) Find an equation for the parabola that has its vertex at the origin and focus $F(0, 6)$.

2) Find an equation for the parabola that has its vertex at the origin and directrix $x = -4$.

3) Find an equation for the ellipse with foci at $(\pm 4, 0)$ and vertices at $(\pm 5, 0)$.

4) Find an equation for the ellipse with length of major axis equal 4, length of minor axis equal 2, and foci on the y-axis.

5) Find an equation for the hyperbola with vertices at $(0, \pm 6)$ and asymptotes $y = \pm\frac{1}{3}x$.

6) Find an equation for the hyperbola with foci at $(0, \pm 3)$ and passing through $(1, 4)$.

7) Complete the square to determine whether the graph of the equation is an ellipse, a parabola, or a hyperbola. If the graph is an ellipse, find the center, foci, vertices, and lengths of the major and minor axes. If it is a parabola, find the vertex, focus, and directrix. If it is a hyperbola, find the center, foci, vertices, and asymptotes. Then sketch the conic section.

   (a) $2x^2 + y^2 = 2y + 1$
   (b) $4x^2 - 4x - 8y + 9 = 0$
   (c) $x^2 + 12 = 4(y^2 + 2x)$

8) Let $z_1 = 1 - 2i$ and $z_2 = 2 + 3i$. Write $z_1 - 2z_2$ and $(1 - i)z_1 + (1 + i)z_2$ in complex standard form $a + bi$.

9) Find the modulus and arguments of $z = 2 - 2\sqrt{3}i$. Write it in polar form.

10) Use De Moirve's theorem to write $(2 - 2\sqrt{3}i)^5$ in both standard form and polar form.

11) Write the four complex values of $\sqrt[4]{15 + 20i}$ in polar form. Approximate them numerically in the standard form. Graph them on the complex plane.

12) Sketch the parametric curve $x = t^3 - 2t$, $y = t^2 - t$ after making a table of values.

Answer key:

1) $x^2 = 24y$

2) $y^2 = 16x$

3) $\frac{x^2}{25} + \frac{y^2}{9} = 1$

4) $\frac{x^2}{1} + \frac{y^2}{4} = 1$

5) $\frac{y^2}{6^2} - \frac{x^2}{18^2} = 1$

6) $\frac{y^2}{8} - \frac{x^2}{1} = 1$

7) (a) Rewrite the equation as $\frac{x^2}{1} + \frac{(y-1)^2}{2} = 1$. This is a shifted ellipse.
   - Center: $(0, 1)$
   - Vertices: $(0, 1 \pm \sqrt{2})$
   - Foci: $(0, 0)$ and $(0, 2)$
   - Length of major axis: $2\sqrt{2}$
   - Length of minor axis: 2

   (b) Rewrite the equation as $\left(x - \frac{1}{2}\right)^2 = 2(y - 1)$. This is a shifted parabola.
   - Vertex: $(\frac{1}{2}, 1)$
   - Focus: $(\frac{1}{2}, \frac{3}{2})$
   - Directrix: $y = \frac{1}{2}$

   (c) Rewrite the equation as $\frac{(x-4)^2}{4} - \frac{y^2}{1} = 1$. This is a shifted hyperbola.
   - Center: $(4, 0)$
   - Vertices: $(2, 0)$ and $(6, 0)$
   - Foci: $(4 \pm \sqrt{5}, 0)$
   - Asymptotes: $y = \frac{1}{2}x - 2$ and $y = -\frac{1}{2}x + 2$

8) $-3 - 8i$ and $-2 + 2i$

9) $|z| = 4$ and $\arg(z) = -\frac{\pi}{3} + k2\pi$. In polar form: $z = 4(\cos(-\pi/3) + i\sin(-\pi/3))$.

10) $z^5 = 1024(\cos(-5\pi/3) + i\sin(-5\pi/3)) = 512 + 512\sqrt{3}i$

11) $\sqrt[4]{15 + 20i} = \sqrt{5}\left(\cos\left(\frac{\theta}{4} + \frac{k2\pi}{4}\right) + i\sin\left(\frac{\theta}{4} + \frac{k2\pi}{4}\right)\right)$ for $k = 0, 1, 2, 3$, where $\theta = \cos^{-1}(\frac{3}{5})$.
   - $k = 0$: $\sqrt{5}\left(\cos\left(\frac{\theta}{4}\right) + i\sin\left(\frac{\theta}{4}\right)\right) \approx 2.1763 + 0.5137\,i$
   - $k = 1$: $\sqrt{5}\left(\cos\left(\frac{\theta}{4} + \frac{2\pi}{4}\right) + i\sin\left(\frac{\theta}{4} + \frac{2\pi}{4}\right)\right) \approx -0.5137 + 2.1763\,i$
   - $k = 2$: $\sqrt{5}\left(\cos\left(\frac{\theta}{4} + \frac{4\pi}{4}\right) + i\sin\left(\frac{\theta}{4} + \frac{4\pi}{4}\right)\right) \approx -2.1763 - 0.5137\,i$
   - $k = 3$: $\sqrt{5}\left(\cos\left(\frac{\theta}{4} + \frac{6\pi}{4}\right) + i\sin\left(\frac{\theta}{4} + \frac{6\pi}{4}\right)\right) \approx 0.5137 - 2.1763\,i$

12) Make a table of values for $t = \pm3$, ±2, ±1, 0.