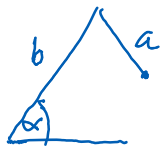
Lecture 10

Friday, May 24, 2024 8:15 AM

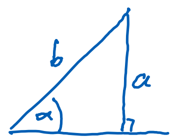
- Law of sines: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

  can be used for the cases ASA, SAA (one side, two angles).

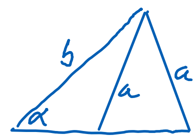
  or SSA (ambiguous case: may get 0, 1, 2 triangles)



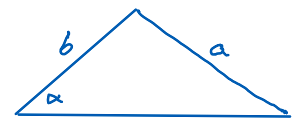
no solution



one solution



two solutions



one solution

- Law of cosines:

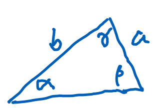
$$\cos A = \frac{b^2+c^2-a^2}{2bc}, \quad \cos B = \frac{c^2+a^2-b^2}{2ac}, \quad \cos C = \frac{a^2+b^2-c^2}{2ab}$$

  can be used for the cases SAS (two sides and the angle in between)

  SSS (three sides)

It is impossible to solve a triangle knowing only 3 angles (no sides).

Ex $a=1$, $b=3$, $\alpha = 30°$



$$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a} = \frac{\sin 30°}{1} = \frac{1}{2}$$

$$\sin\beta = \frac{b}{2} = \frac{3}{2} > 1 \quad \text{(impossible)}$$

Ex $a = \frac{3}{2}$, $b = 3$, $\alpha = 30°$

$$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a} = \frac{1/2}{3/2} = \frac{1}{3}$$

$$\rightsquigarrow \sin\beta = \frac{b}{3} = 1 \rightsquigarrow \beta = 90° \rightsquigarrow \gamma = 180° - 90° - 30° = 60°$$

$$c = \frac{\sin\gamma}{\sin\alpha}\, a = \frac{\sin 60°}{\sin 30°}\, \frac{3}{2} = \frac{3\sqrt{3}}{2}$$

Ex $a = 2$, $b = 3$, $\alpha = 30°$

$$\sin\beta = \frac{\sin\alpha}{a} b = \frac{1/2}{2} 3 = \frac{3}{4}$$

$$\beta \approx 48.59° \text{ or } \beta = 180° - 48.59° = 131.41°$$

For each case, we compute γ and c.

So, there will be two triangles.

Ex $a = 4$, $b = 3$, $\alpha = 30°$

$$\sin\beta = \frac{\sin\alpha}{a} b = \frac{1/2}{4} 3 = \frac{3}{8}$$

$\leadsto$ There are two choices for β: $\beta = \underbrace{\arcsin\left(\frac{3}{8}\right)}_{\approx 22.02°}$, or $\beta = \underbrace{180° - \arcsin\left(\frac{3}{8}\right)}_{\approx 157.98°}$.

For $\beta \approx 22.02°$, $\gamma = 180° - \alpha - \beta = 180° - 30° - 22.02° = 127.98°$

$$\leadsto c = \frac{\sin\gamma}{\sin\alpha} a = \dots$$

For $\beta \approx 157.98°$, $\gamma = 180° - \alpha - \beta = 180° - 30° - 157.98° < 0$: impossible

* Some examples of the law of cosines

* The theodolite instrument for surveying.

* Trigonometric identities:

Prove that $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

$$\frac{1 - \sin x}{1 + \sin x} = (\sec x - \tan x)^2$$