Lecture 11

Wednesday, May 29, 2024 1:30 AM

Chapter 7: Trigonometric identities

We are leaving the comfort of having a picture (unit circle, triangle), and will deal with algebra.

* Pythagorean identities:

$$\cos^2 x + \sin^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

* Even/odd identities:

$$\sin(-x) = -\sin x$$

$$\cos(-x) = \cos x$$

$$\tan(-x) = -\tan x$$

* Co-function identities (or complementary-angle identities):

$x + y = \frac{\pi}{2}$: x, y are called complementary to each other

$$\sin x = \cos y$$

$$\cos x = \sin y$$

$$\tan x = \cot y$$

$$\cot x = \tan y$$

$$\sec x = \csc y$$

$$\csc x = \sec y$$

* Supplementary-angle identities:

$x + y = \pi$: x, y are supplementary of each other

$\sin x = \sin y$

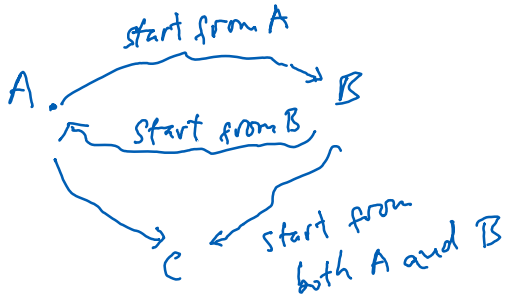
$\cos x = -\cos y$

$\tan x = -\tan y$

Ex Show that $\sin(x + \frac{\pi}{2}) = \cos x$ [for any value of x.]

Here we are not solving for x. We are proving that the equation is true for any value of x. It is not enough to show that the eq. is true for only a few values of x.

Showing $A = B$:



$$\sin\left(x + \frac{\pi}{2}\right) = \cos(-x) = \cos x$$

$=$: co-function; $=$: even/odd

Ex Show that

$$\frac{\cos x}{\sec x \sin x} = \csc x - \sin x$$

Turn everything into sine and cosine.

LHS = ...

RHS = ...

Ex Show that $\dfrac{\sin x + \cos x}{\sec x + \csc x} = \sin x \cos x$

Ex Show that $(\tan x + \cot x)^2 = \sec^2 x + \csc^2 x$