Lecture 14

Wednesday, June 5, 2024 8:43 AM

Trigonometric equations

Equation : not true for every x ← solve

Identity : true for all x ← prove

$x^2 = 5$ is an equation, not identity

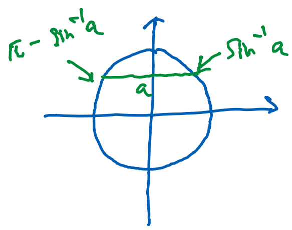
$x^2 = x^2$ is an identity

To solve an equation is to find all values of x that satisfy the equation.

We'll start from the basic equations:

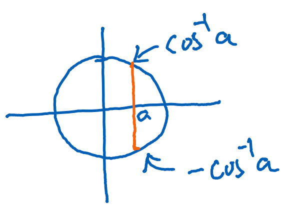
$$\left.\begin{aligned} \sin x &= a \\ \cos x &= a \\ \tan x &= a \end{aligned}\right\}$$ these are the 3 building blocks of trigonometric equations

The first step is to find all solutions in one period. Then add multiples of the period to get all solutions.



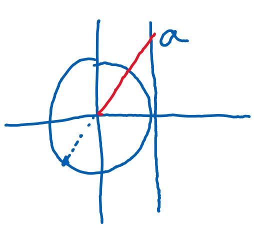
$\sin x = a$ has two solutions in one period:

$x = \sin^{-1} a$ and $x = \pi - \sin^{-1} a$



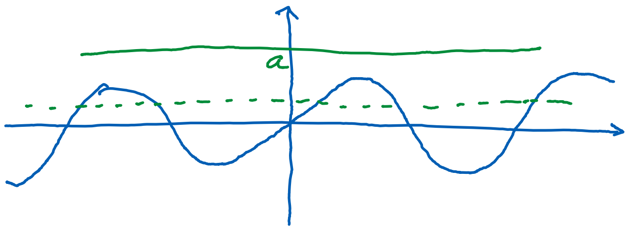
$\cos x = a$ has two solution in one period:

$x = \cos^{-1} a$ and $x = -\cos^{-1} a$



$\tan x = a$ has one solution in one period:

$x = \tan^{-1} a$



$\sin x = a$ either has no solutions or has ∞-many solutions

$*$ All sols to $\sin x = a$ are:

- If $a > 1$ or $a < 1$: no sols
- If $-1 \le a \le 1$:

$$x = \sin^{-1} a + k2\pi$$
$$x = \pi - \sin^{-1} a + k2\pi$$
(k is any integer)

$*$ All sols to $\cos x = a$ are:

- If $a > 1$ or $a < 1$: no sols
- If $-1 \le a \le 1$:

$$x = \pm\cos^{-1} a + k2\pi$$
(k is any integer)

$*$ All sols to $\tan x = a$ are:

$$x = \tan^{-1} a + k\pi$$
(k is any integer)

$\underline{\underline{\text{Ex}}}$ Solve the equation $\cot x = \sqrt{3}$

$\underline{\underline{\text{Ex}}}$ Solve the equation $\sin x = \frac{\sqrt{2}}{2}$ for $x \in [9\pi, 12\pi]$

$$x = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) + k2\pi = \frac{\pi}{4} + k2\pi$$

$$x = \pi - \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) + k2\pi = \frac{3\pi}{4} + k2\pi$$

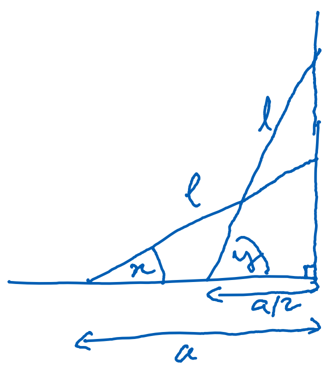
k	$\frac{\pi}{4} + k2\pi$	$\frac{3\pi}{4} + k2\pi$
4	$\frac{\pi}{4} + 8\pi$	$\frac{3\pi}{4} + 8\pi$
5	$\frac{\pi}{4} + 10\pi$ ✓	$\frac{3\pi}{4} + 10\pi$ ✓
6	$\frac{\pi}{4} + 12\pi$	$\frac{3\pi}{4} + 12\pi$

Ex: Solve the equation $\cos x(\sin x + \frac{1}{2}) = 0$

for $x \in [0, 2\pi]$.

k	$\frac{\pi}{2} + k2\pi$	$-\frac{\pi}{2} + k2\pi$
0	$\frac{\pi}{2}$ ✓	$-\frac{\pi}{2}$
1	$\frac{\pi}{2} + 2\pi$	$\frac{3\pi}{2}$ ✓
2	$\frac{\pi}{2} + 4\pi$	$\frac{7\pi}{2}$

Ex Ladder problem



$\cos x = \frac{a}{\ell}$, $\cos y = \frac{a/2}{\ell}$

Thus, $\cos x = 2\cos y$

For $y = 2x$, we need $\cos x = 2\cos 2x$

Double-angle identity: $\cos 2x = 2\cos^2 x - 1$

Need to solve:

$$\cos x = 2(2\cos^2 x - 1)$$

Let $t = \cos x \in [-1, 1]$:

$$t = 2(2t^2 - 1)$$

$\rightsquigarrow$ $4t^2 - t - 2 = 0$

$\rightsquigarrow$ solve for t

$\rightsquigarrow$ for each t, solve for x from $\cos x = t$.