Lecture 15

Friday, June 7, 2024 4:19 PM

Continue the ladder problem:

$$\cos x = 2\cos 2x, \quad 0 < x < 90^\circ$$

$t = \cos x$:

$$t = 2(2t^2 - 1) \rightsquigarrow 4t^2 - t - 2 = 0 \rightsquigarrow t = \frac{1 \pm \sqrt{33}}{8}$$

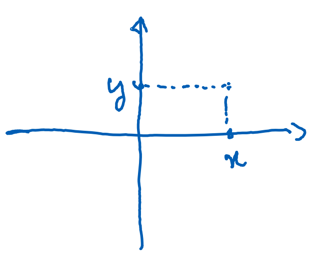
Knowing that $\cos x > 0$, we only choose $t = \dfrac{1+\sqrt{33}}{8}$.

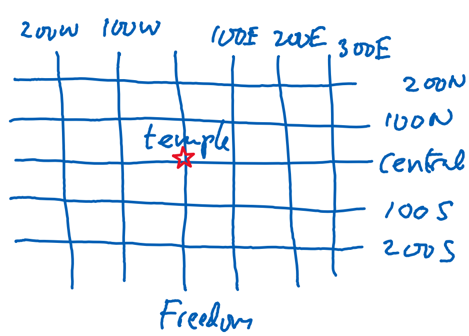
$$\cos x = \frac{1+\sqrt{33}}{8}$$

$$\rightsquigarrow x = \cos^{-1}\left(\frac{1+\sqrt{33}}{8}\right) \approx 32.52^\circ$$

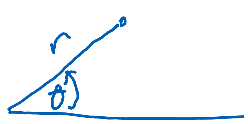
Polar coordinates

Rectangular coordinates (Cartesian coords) : (x, y)





Provo City

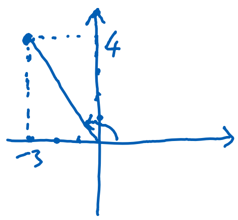


(r, θ): polar coordinates

Conversion to rectangular coordinates : $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

Ex Convert $(r,\theta) = (1, \frac{\pi}{3})$ to rectangular coords

Ex Convert $(x,y) = (-3, 4)$ to polar coords

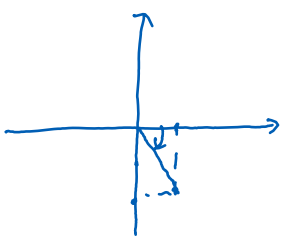


$r = 5$

$\cos\theta = \frac{x}{r} = -\frac{3}{5}$

$\theta = \cos^{-1}\left(-\frac{3}{5}\right) + k2\pi$

Ex Convert $(x,y) = (1, -\sqrt{3})$ to polar coords

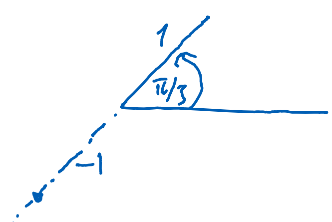


$r = 2$, $\cos\theta = \frac{x}{r} = \frac{1}{2}$

$\theta = -\cos^{-1}\left(\frac{1}{2}\right) + k2\pi = -\frac{\pi}{3} + k2\pi$

So far, we require $r > 0$. We will now relax this condition and allow $r < 0$.

$(r,\theta) = (-1, \frac{\pi}{3})$:



This is equivalent to the polar coords $(1, \frac{4\pi}{3})$.