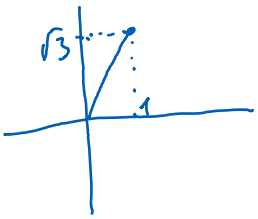
Lecture 16

Monday, June 10, 2024 1:15 PM

Ex find polar coords of $(1,\sqrt{3})$, with $\begin{cases} r>0 \\ r<0 \end{cases}$



$r = \sqrt{x^2+y^2} = \sqrt{2}$

$\cos\theta = \frac{x}{r} = \frac{1}{2} \rightsquigarrow \theta = \frac{\pi}{3}$

Thus, $(r,\theta) = \left(\sqrt{2}, \frac{\pi}{3}\right)$ or $\left(-\sqrt{2}, \frac{\pi}{3}+\pi\right) = \left(-\sqrt{2}, \frac{4\pi}{3}\right)$

Polar equations

A polar eq. is any eq. involving r and θ, such as

$\theta = 5$

$r = 2$

$\theta r = 1$

$r = \sin(e^{\theta})$

....

A Cartesian eq. is any eq. involving x and y, such as

$x+y=0$

$y = \sin x$

$x^2+y^2 = 2x+1$

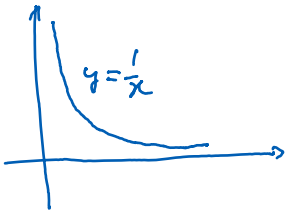
.....

How do we convert an equation from Cartesian coords to polar coords?

Ex: $x^2+y^2=1$ (unit circle)

$\rightsquigarrow r^2 = 1 \rightsquigarrow r = 1$ (or $r=-1$)

Ex



$$r\sin\theta = \frac{1}{r\cos\theta}$$

$$\rightsquigarrow r^2 \sin\theta\cos\theta = 1$$

$$\rightsquigarrow r^2 2\sin\theta\cos\theta = 2$$

$$\rightsquigarrow r^2\sin(2\theta) = 2 \rightsquigarrow r^2 = 2\csc(2\theta)$$

$$\rightsquigarrow r = \sqrt{2\csc(2\theta)}$$

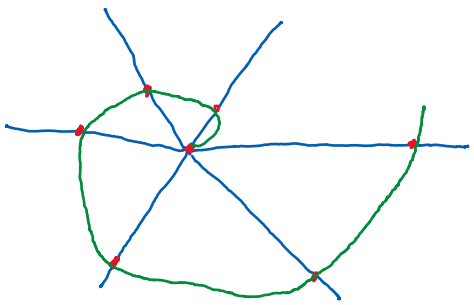
Ex $x=1$ (vertical line)

$$r\cos\theta = 1 \rightsquigarrow r = \frac{1}{\cos\theta} = \sec\theta$$

Plot polar equations

Ex: $r=\theta$

θ	r
0	0
1	1
2	2
3	3
4	4
5	5

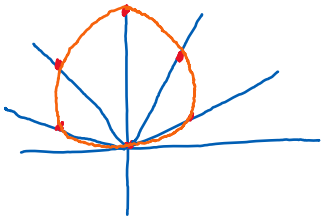


Spiral

Ex $r=\sin\theta$

θ	r
0	0
$\pi/6$	$1/2$
$\pi/3$	$\sqrt{3}/2$
$\pi/2$	1
$\pi/2+\pi/6$	$\sqrt{3}/2$
$\pi/2+\pi/3$	$1/2$
π	0

θ	r
$\pi+\frac{\pi}{6}$	$-1/2$
$\pi+\frac{\pi}{3}$	$-\sqrt{3}/2$
$\pi+\frac{\pi}{2}$	-1
$\frac{3\pi}{2}+\frac{\pi}{6}$	$-\sqrt{3}/2$
$\frac{3\pi}{2}+\frac{\pi}{3}$	$-1/2$
2π	0



Use Geogebra to sketch polar equations:

https://www.geogebra.org/m/upbPEhNK
https://www.geogebra.org/m/ApcfSCZY

Practice with matching problems:

https://janpowell.weebly.com/uploads/5/4/4/3/54437167/unit_4_polar_graphs.pdf