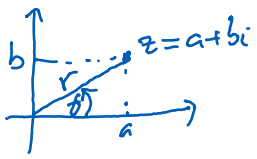
Lecture 18

Thursday, June 13, 2024 1:18 PM

Polar form of complex numbers:



$r = \sqrt{a^2+b^2}$ is called the modulus or absolute value of z.

Notation $r = |z|$.

θ is called the argument of z. Note that θ is only defined up to a multiple of 2π.

$$z = a+bi = r\cos\theta + ir\sin\theta = r(\cos\theta + i\sin\theta)$$

$$z_1 z_2 = r_1(\cos\theta_1 + i\sin\theta_1)\, r_2(\cos\theta_2 + i\sin\theta_2) = r_1 r_2(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

Rule: to multiply two complex numbers, we multiply the modulus and add the arguments.

Ex: $z = 3+4i$

Find z^{10}.

$$z = 5\left(\frac{3}{5} + \frac{4}{5}i\right) = 5(\cos\theta + i\sin\theta), \quad \theta \approx \dots$$

$$z^{10} = 5^{10}(\cos 10\theta + i\sin 10\theta)$$

Taking n'th roots:

$$\sqrt[n]{z} = w$$

$$w^n = z = r(\cos\theta + i\sin\theta)$$

$$w = s(\cos\omega + i\sin\omega) \rightsquigarrow w^n = s^n(\cos n\omega + i\sin n\omega)$$

$$\rightsquigarrow s = \sqrt[n]{r}, \quad n\omega = \theta + k2\pi \rightsquigarrow \omega = \frac{\theta+k2\pi}{n}.$$