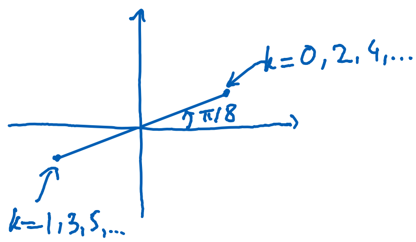
Lecture 19

Monday, June 17, 2024 4:30 PM

Recall:

$$\sqrt{1+i} = \sqrt[4]{2}\left(\cos\left(\frac{\pi}{8}+k\pi\right) + i\sin\left(\frac{\pi}{8}+k\pi\right)\right)$$



How do we compute $\sqrt[3]{1+i}$?

$$z = \sqrt[3]{1+i}$$

$$z^3 = 1+i$$

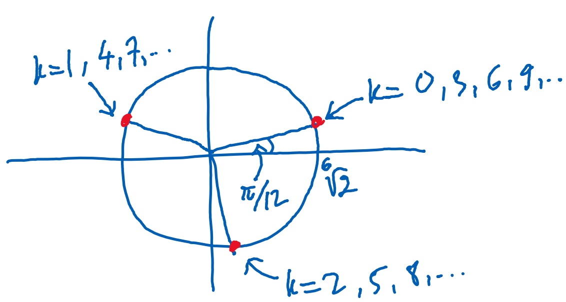
Write in polar form: $z = r(\cos\theta + i\sin\theta)$

$$\rightsquigarrow z^3 = r^3(\cos 3\theta + i\sin 3\theta)$$

$$1+i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

want $\begin{cases} r^3 = \sqrt{2} \\ 3\theta = \frac{\pi}{4} + k2\pi \end{cases} \rightsquigarrow \begin{cases} r = \sqrt[6]{2} \\ \theta = \frac{\pi}{12} + k\frac{2\pi}{3} \end{cases}$

Therefore, $\sqrt[3]{1+i} = \sqrt[6]{2}\left(\cos\left(\frac{\pi}{12}+k\frac{2\pi}{3}\right) + i\sin\left(\frac{\pi}{12}+k\frac{2\pi}{3}\right)\right)$



More generally, if $z = r(\cos\theta + i\sin\theta)$ then

$$\sqrt[n]{z} = \sqrt[n]{r}\left(\cos\left(\frac{\theta}{n} + k\frac{2\pi}{n}\right) + i\sin\left(\frac{\theta}{n} + k\frac{2\pi}{n}\right)\right), \quad k = 0, 1, \dots, n-1$$

Each non-zero complex number has exactly n n'th roots. They are equally spaced on the circle of radius $\sqrt[n]{r}$.

Ex find $\sqrt[8]{-256}$

Parametric equation of a curve

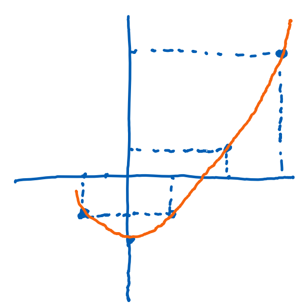
$(x(t), y(t))$ - a pair of function

For each t, we get a point on the plane with coords $x(t)$ and $y(t)$.

Ex $(x,y) = (2t, t^2 - 3)$

defines a curve.

t	(x,y)
-1	$(-2,-2)$
0	$(0,-3)$
1	$(2,-2)$
2	$(4,1)$
3	$(6,6)$



Use Geogebra to sketch a parametric curve:

https://www.geogebra.org/m/yJNhQMQa