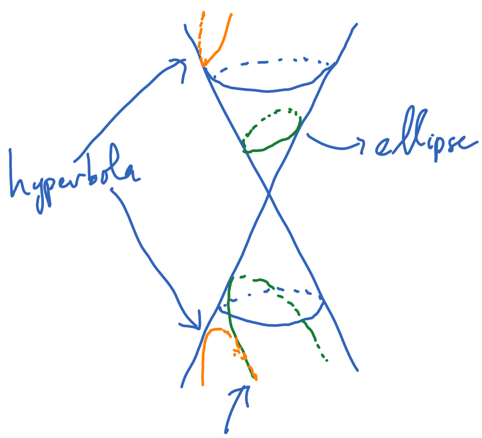
Lecture 20

Friday, June 21, 2024 12:49 AM

Three conic sections:
- ellipse (circle being a special case)
- parabola
- hyperbola

These conic sections are very rich in geometric properties.

Imagine an infinite double cone:



parabola (parallel to a generator of the cone)

Imagine that you cut it by an infinite plane: if the cross section is a finite curve, that is an ellipse. If the plane is parallel to a generator, that is a parabola. Otherwise, it is a hyperbola.

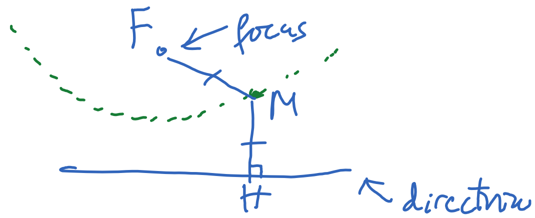
* Another way to interpret conic sections:

parabola	ellipse	hyperbola
focus, directrix	focus, focus	focus, focus
	focus, directrix	focus, directrix

will learn in Calc II (Math 213)

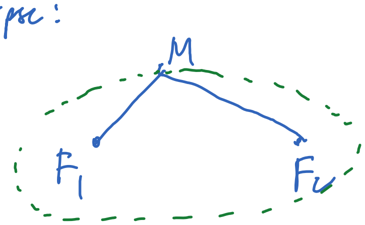
We will view conic sections as geometric objects, not as graph of functions.

* Parabola:



parabola is the collection of points M such that $MF = MH$.

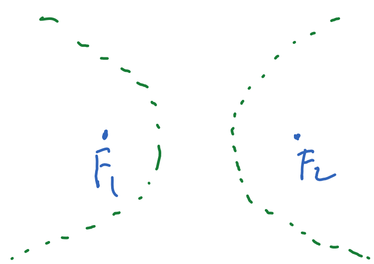
* Ellipse:



$MF_1 + MF_2 = 2a > 0$

constant

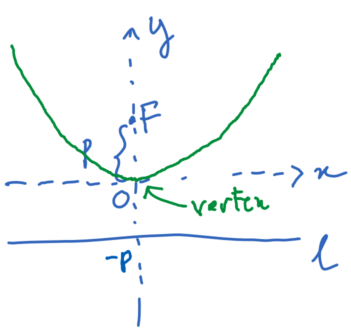
* Hyperbola:



$MF_2 - MF_1 = 2a$

$MF_1 - MF_2 = 2a$

Equations on the Cartesian coordinates.



$F(0, p)$

$l : y = -p$

$x^2 = 4py$

We choose a coordinate system that makes the equation of the parabola as simple as possible.

Ex: Find the equation of a parabola with focus $F(0,1)$ and vertex $(0,0)$.

Ex vertex $(0,0)$, directrix $y=\frac{1}{2}$.