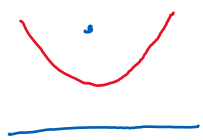
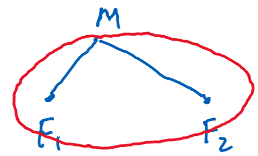
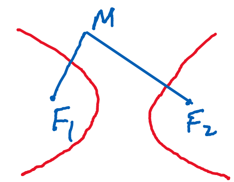
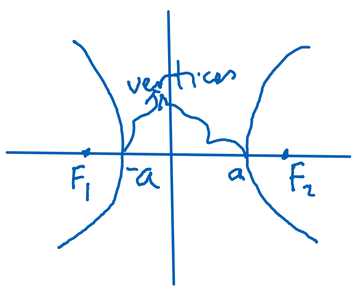
Lecture 21

Monday, June 24, 2024 9:06 AM

parabola	ellipse	hyperbola
focus, directrix	focus, focus	focus, focus
		
	$MF_1+MF_2=2a$	$\lvert MF_1-MF_2\rvert=2a$

* Hyperbola:



With this choice of coordinate system, the hyperbola has an equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Vertices: $(\pm a, 0)$

Foci: $(\pm c, 0)$, where $c^2=a^2+b^2$

Does the hyperbola look like a parabola?

The answer is no. As x and y get large, the hyperbola looks like a line (more precisely, a pair of lines) called asymptotes. The parabola has no asymptotes.

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \rightsquigarrow \frac{1}{x^2}\left(\frac{x^2}{a^2}-\frac{y^2}{b^2}\right)=\frac{1}{x^2}$$

$$\rightsquigarrow \left(\frac{1}{a}\right)^2-\left(\frac{y}{bx}\right)^2=\frac{1}{x^2}\approx 0$$

So, $\left(\frac{1}{a}\right)^2 \approx \left(\frac{y}{bx}\right)^2 \rightsquigarrow \pm\frac{1}{a} \approx \frac{y}{bx} \rightsquigarrow y \approx \pm\frac{b}{a}x$.

As x gets large, y is approximately $\pm\frac{b}{a}x$.

The hyperbola grows at a linear rate, whereas the parabola grows at a quadratic rate (thus, faster).

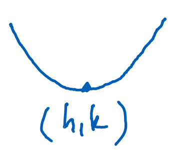
Ex Find the vertices, foci, and asymptotes of the following hyperbola

(a) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

(b) $4x^2 - y^2 = 9$

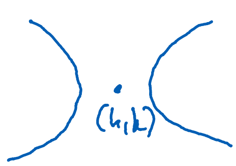
(c) $8y^2 - 2x^2 = 2$

Shifted conic sections



$(x-h)^2 = 4p(y-k)$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$



$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Ex Find the vertices, foci, and asymptotes of the hyperbola

$$x^2+4y^2-2x+8y=0$$

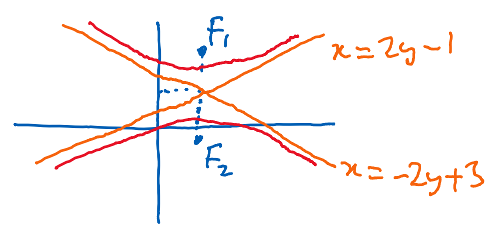
$$(x^2-2x)-(4y^2-8y)=0$$

$$\rightsquigarrow [(x-1)^2-1]-4(y^2-2y)=0$$

$$\rightsquigarrow [(x-1)^2-1]-4[(y-1)^2-1]=0$$

$$\rightsquigarrow (x-1)^2-4(y-1)^2=-3 \rightsquigarrow 4(y-1)^2-(x-1)^2=3$$

$$\rightsquigarrow \frac{4(y-1)^2}{3}-\frac{(x-1)^2}{3}=1$$



$$\rightsquigarrow \frac{(y-1)^2}{3/4}-\frac{(x-1)^2}{3}=1$$

$$\rightsquigarrow a=\sqrt{3/4}=\sqrt{3}/2, \quad b=\sqrt{3}$$

$$c=\sqrt{a^2+b^2}=\sqrt{\frac{3}{4}+3}=\frac{\sqrt{15}}{4}$$

Center of the hyperbola is $(1,1)$.

The vertices are $(1\pm a, 1)=(1\pm\frac{\sqrt{3}}{2}, 1)$

The foci are $(1\pm c, 1)=(1\pm\frac{\sqrt{15}}{4}, 1)$

The asymptotes are $x-1=\pm\frac{b}{a}(y-1)=\pm 2(y-1)$

The first asymptote is $x-1=2(y-1)=2y-2$, which is equivalent to

$$x=2y-1$$

The second asymptote is $x-1=-2(y-1)$, which is equiv. to $x=-2y+3$