Lecture 3

Monday, May 6, 2024 9:00 AM

Trigonometric functions of special values of t

t	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
$\sin t$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
$\cos t$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1
$\tan t$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	X	0
$\cot t$	X	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	X
$\sec t$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	X	-1
$\csc t$	X	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	X

remember these values

$$\tan t = \frac{\sin t}{\cos t}$$

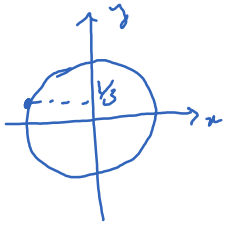
$$\cot t = \frac{1}{\tan t}$$

$$\sec t = \frac{1}{\cos t}$$

$$\csc t = \frac{1}{\sin t}$$

$$\cos^2 t + \sin^2 t = 1$$

Ex If $\sin t = \frac{1}{3}$ and t is in Quadrant II, find $\cos t$, $\tan t$, $\cot t$, $\sec t$, $\csc t$.



$$\cos^2 t = 1 - \sin^2 t = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\cos t = -\sqrt{\frac{8}{9}} = -\frac{2\sqrt{2}}{3}$$

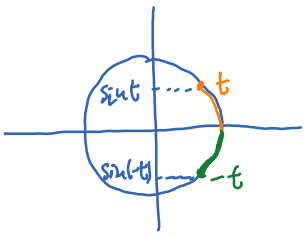
$$\tan t = \frac{\sin t}{\cos t} = \frac{1/3}{-\frac{2\sqrt{2}}{3}} = -\frac{1}{2\sqrt{2}}$$

$$\cot t = \frac{1}{\tan t} = -2\sqrt{2}$$

$$\sec t = \frac{1}{\cos t} = -\frac{3}{2\sqrt{2}}$$

$$\csc t = \frac{1}{\sin t} = 3$$

Even-odd properties:



$\sin(-t) = -\sin t$

$\cos(-t) = \cos t$

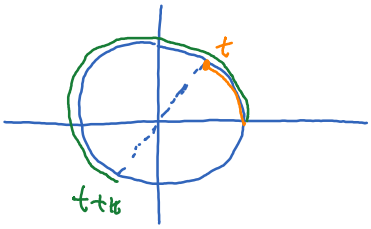
$\tan(-t) = -\tan t$

$\cot(-t) = -\cot t$

$\sec(-t) = \sec(t)$

$\csc(-t) = -\csc t$

Shifting by π:



$\sin(t+\pi) = -\sin t$

$\cos(t+\pi) = -\cos t$

$\tan(t+\pi) = \tan \pi$

$\cot(t+\pi) = \cot(t)$

$\sec(t+\pi) = -\sec t$

$\csc(t+\pi) = -\csc(t)$.

How to use calculator to compute trigonometric functions? make sure to choose the radian mode instead of degree mode.

Ex $\sin(2.2)$, $\cos(2.5)$, $\tan\left(\frac{\pi}{8}\right)$, $\cot\left(\frac{\pi}{8}\right)$, $\sec\left(\frac{\pi}{8}\right)$, $\csc\left(\frac{\pi}{8}\right)$.

Graph of trigonometric functions

$\sin x$, $\cos x$ — defined everywhere

$\tan x$, $\cot x$, $\sec x$, $\csc x$ — have vertical asymptotes

* Periodicity: sine and cosine are periodic with period 2π

$\sin(x+2\pi) = \sin x$

$\cos(x+2\pi) = \cos x$

$\sin(x), \cos(x)$ are always in between -1 and 1

* Graph sine and cosine using Geogebra.

- Compare the graphs of $\sin x$, $\sin 2x$, $\sin 3x$, $\sin 4x$, ...

  What can you say about the graph of $\sin(kx)$?

- Compare the graphs of $\sin x$, $2\sin x$, $3\sin x$, $4\sin x$, ...

  What can you say about the graph of $a \sin x$?

- Compare the graphs of $\sin x$, $\sin x + 1$, $\sin x + 2$, $\sin x + 3$, ...

  What can you say about the graph of $\sin x + b$?

- Compare the graph of $\sin(x)$, $\sin(x+1)$, $\sin(x+2)$

  What can you say about the graph of $\sin(x+c)$?