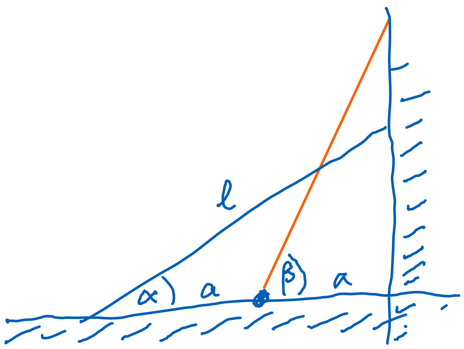
Lecture 8

Monday, May 20, 2024 8:47 AM

Ladder problem:



Will the angle α be doubled?

$\cos\alpha = \dfrac{2a}{l}$, $\cos\beta = \dfrac{a}{l}$

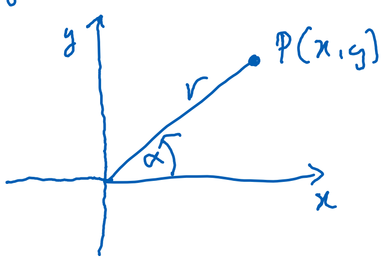
So, $\cos\alpha = 2\cos\beta$

For $\alpha = 30^\circ$, compare β with 2α

For $\alpha = 35^\circ$, compare β with 2α

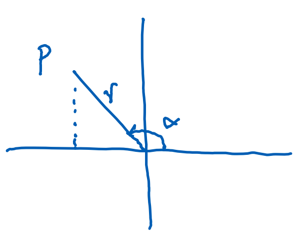
Later, when you learn the double-angle identity, you will see that only for $\alpha \approx 32.53^\circ$ will $\beta = 2\alpha$.

- Trigonometric functions of angles (now angles may not be acute):



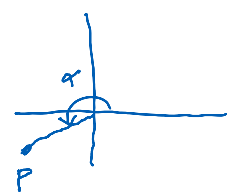
$\sin\alpha = \dfrac{y}{r}$ $\tan\alpha = \dfrac{y}{x}$ $\sec\alpha = \dfrac{r}{x}$

$\cos\beta = \dfrac{x}{r}$ $\cot\alpha = \dfrac{x}{y}$ $\csc\alpha = \dfrac{r}{y}$



$\cos\alpha < 0$

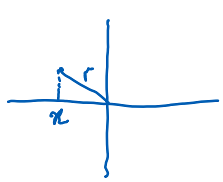
$\sin\alpha > 0$



$\cos\alpha < 0$

$\sin\alpha < 0$

Ex Find $\tan\theta$, where θ is in quadrant II and $\cos\theta = \dfrac{-1}{4}$.



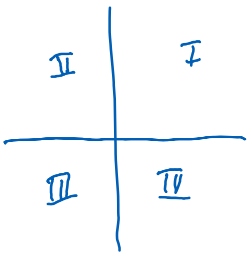
$\cos\theta = \dfrac{x}{r} = -\dfrac{1}{4}$

Choose $x = -1$ and $r = 4$. Then $y = \sqrt{4^2 - (-1)^2} = \sqrt{15}$

Then $\tan\theta = \frac{y}{x} = \frac{\sqrt{15}}{-1} = -\sqrt{15}$

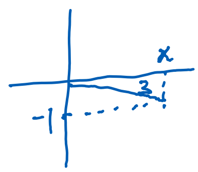
$\underline{\underline{\text{Ex}}}$ Find $\tan\left(\sin^{-1}\left(-\frac{1}{3}\right)\right)$

Let $\theta = \sin^{-1}\left(-\frac{1}{3}\right)$. Then $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ and $\sin\theta = -\frac{1}{3}$.



So, θ is in the IV quadrant.

Choose $y=-1$ and $r=3$

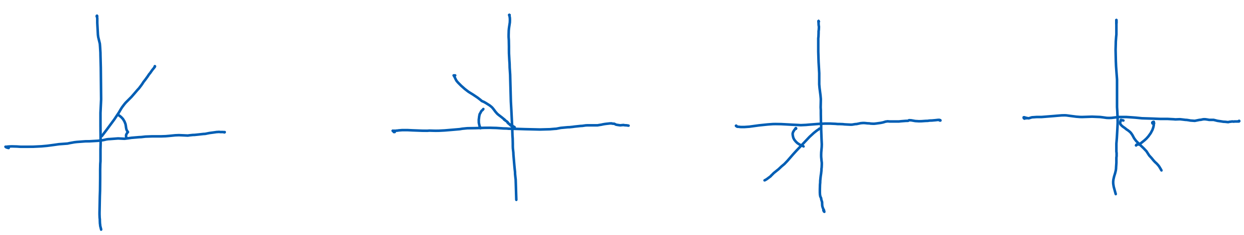


By Pythagorean thm:

$x = \sqrt{3^2-(-1)^2} = \sqrt{8} = 2\sqrt{2}$

Therefore, $\tan\theta = \frac{y}{x} = \frac{-1}{2\sqrt{2}}$

<u>Reference angle</u>



$$\bar{\theta} = \left|\operatorname{round}\left(\frac{\theta}{\pi}\right) - \frac{\theta}{\pi}\right|\pi \quad \text{(in radian)}$$

$$\bar{\theta} = \left|\operatorname{round}\left(\frac{\theta}{180}\right) - \frac{\theta}{180}\right|180 \quad \text{(in degree)}$$