

Lab 1

You don't need to have prior programming experience to be able to do this lab, although such experience will be helpful. *The next lab will be built upon this lab, so please make sure that you go through all the instruction carefully and do all the assignments.* Below, you will learn how to:

- do basic arithmetic on numbers,
- define functions and graph them,
- do basic operations on vectors.

1 To turn in

Do Problems 1-35 in a single Mathematica Notebook file, or ipynb file if you use JupyterLab. Write *your name* and *lab number* at the beginning of your report. Clearly label each problem to separate them from each other. Make sure to comment on each problem. If your code doesn't run correctly, explain what you are trying to do. Submit on Canvas both the *pdf file* and the *source file* (nb or ipynb). I will re-run all commands when I grade. Here is the breakdown of points:

Problems	Points
1-3, 6, 7, 9-13, 16-19, 21-28, 31, 32, 34	1
4, 5, 8, 14, 15, 20, 29	2
30, 33, 35	3
Readability of your report	3
Total: 35	Total: 51

2 Basic arithmetic

- (1) Type $(1+2)^2-3*a$ followed by **Shift+Enter**.
- (2) Type $a=5$ followed by **Enter**. Then type $(1+2)^2-3*a$ followed by **Shift+Enter**.
- (3) Type $35/6$, then **Shift+Enter**. Do it give you a fraction or a decimal-point number?
- (4) Type $(1+2)^2$ followed by **Shift+Enter**. Then type $[1+2]^2$ followed by **Shift+Enter**. What can you learn from this experiment?
- (5) Type $N[35/6]$ followed by **Shift+Enter**. Then type $N[35/6,10]$ followed by **Shift+Enter**. Then type $N[35/6,20]$ followed by **Shift+Enter**. What does the command $N[...]$ do?
- (6) The number $\sqrt{2}$ in Mathematica is `Sqrt[2]` (notice the capitalized S). Type `Sqrt[2]` followed by **Shift+Enter**. Does it give you a decimal-point number?
- (7) Type $N[Sqrt[2]]$ followed by **Shift+Enter**. An equivalent way of writing this is `Sqrt[2] // N`
- (8) Find $\sqrt{2}$ using 10 significant digits.
- (9) Type `Sin[Pi]` followed by **Shift+Enter**. Next, try the same command but with lowercase S and/or P. Is Mathematica case sensitive?
- (10) Type 47^{100} ; (with the semicolon) followed by **Shift+Enter**. Then type 47^{100} (without semicolon) followed by **Shift+Enter**.

The semicolon is to hold the output. One uses it when output is too long or not of interest. You may have noticed that the function `N` is to evaluate a numerical value of an expression. Each function's name is capitalized is followed by a pair of square brackets `[...]` instead of parentheses (...) as we normally write on paper. For example, the function $\sin(x)$, $\cos(x)$, $\tan(x)$, $\exp(x)$, $\ln(x)$ are written as `Sin[x]`, `Cos[x]`, `Tan[x]`, `Exp[x]`, `Log[x]` respectively in Mathematica.

- (11) The number e is typed as `Exp[1]` or `E`. Find the value of e using 10 significant digits.
- (12) Find the value of $\ln 2$ using 10 significant digits.
- (13) Use a suitable property of logarithm to find $\log_2 3$ using 10 significant digits.
- (14) Find a numerical value of $e^{2(\cos(\sqrt{2})+\sin(5))} + \ln 2$. Be careful to use square brackets and parentheses properly.

3 Define and graph functions

To define a function, we use a combination of the underscore sign “_” and the operator “:=” as follows.

- (15) `f[x_] := Sin[x]+Cos[x]` (notice the underscore after `x`) followed by `Shift+Enter`. Then type `f[Pi]+f[Pi/4]` followed by `Shift+Enter`. Can you find a numerical value of this number?
- (16) Try dropping the underscore after x and repeat the previous exercise to see what happens.

Note that the underscore is required only when *defining* a function. It is not needed when *using* the function.

- (17) Define two functions $f(x) = \ln x$ and $g(x) = \tan(\cos x)$. Then evaluate numerically the quantities $f(f(10))$ and $f(1 + g(\frac{\pi}{8}))$.
- (18) To clear a quantity from the memory, use the command `Clear`. Type `Clear[f]` followed by `Shift+Enter`. Then type `f[Pi]+f[Pi/4]` followed by `Shift+Enter`. What do you observe?

To define a function of more than one variable, for example $f(x, y) = x^2 - xy + y^3$, we do as follows.

- (19) Type `f[x_,y_] := x^2 - x*y + y^3` followed by `Shift+Enter`. Make sure that within the square bracket you put an underscore after each variable.
Warning: the product xy is typed as either `x*y` or `x y` (space between `x` and `y`). If you type `xy`, Mathematica will understand `xy` as a single variable rather than the product of `x` and `y`.
- (20) Define the function $g(x, y, z) = \frac{1}{x^2+xyz}$ and then compute numerically $f(3, 2) + g(1, 2, -1)$.

Now let us plot functions of one variable. Try the following command:

- (21) `Plot[Sin[x]+Cos[2x], {x,0,2*Pi}]`, then `Shift+Enter`.
- (22) To customize the style of the graph, you can add the option `PlotStyle` to the `Plot` command. Try the following:

```
Plot[Sin[x]+Cos[2x], {x,0,2*Pi}, PlotStyle -> {Red, Dashed}]
```

Then `Shift+Enter`. Note that the arrow is typed as `->`.

(23) You can also give the function a name before plotting it. For example,

```
f[x_] := Sin[x]+Cos[2x];  
Plot[f[x], {x,0,2*Pi}, Filling -> Axis]
```

Then **Shift+Enter**. In the second command, note that the underscore within the brackets is no longer needed because f was already defined.

(24) To graph two functions, say $\sin(x)$ and $\cos(x)$, on the same plot, we do the following:

```
Plot[{Sin[x],Cos[x]},{x,0,5*Pi}]
```

To graph $\sin(x)$ with dashed line with green color, and graph $\cos(x)$ with thin line with red color, do the following:

```
Plot[{Sin[x],Cos[x]},{x,0,5*Pi}, PlotStyle->{{Dashed,Green},{Thin,Red}}]
```

To label the graphs, add the option `PlotLegends->Automatic` into the `Plot` command.

Next, let us plot functions of two variables, for example

$$f(x,y) = x^2 \sin(y)$$

We will use the command **Plot3D** instead of **Plot**. Try the following commands:

(25) `Plot3D[x^2*Sin[y], {x,-2,2}, {y,-Pi,Pi}]` followed by **Shift+Enter**.

You can rotate the above surface by dragging your mouse. If you hold the **Ctrl** key while clicking on the picture and dragging up/down you will zoom in/out of the picture. Holding the **Shift** key while dragging will move the picture around the window. Note that if you use JupyterLab, you will have to put the prefix `Interact@` in front of `Plot3D` to be able to use this feature.

(26) You can also give the function a name before plotting it. Try the following:

```
f[x_,y_] := x^2*Sin[y];  
Plot[f[x,y], {x,-2,2}, {y,-Pi,Pi}]
```

There are many interesting options you can use for 3D graphing. Check out [Wolfram's webpage](#).

4 Basic operations on vectors

A vector is an ordered list of numbers. In Mathematica, a list is enclosed between a pair of curly brackets `{}`. For example, to define vectors $a = (1, 2, 3)$ and $b = (1, 0, -1)$, we write

(27) `a={1,2,3}`
`b={1,0,-1}`

and then **Shift+Enter**. Note that you must use the curly brackets `{...}`, not square brackets `[...]` nor the parentheses `(...)`.

(28) You can add, subtract, scale vectors. You also take the dot product of two vectors. Try the following:

```
a+b
2a
2a-3b
a.b
(a-b).(-a+2b)
```

then **Shift+Enter**.

- (29) To solve an equation, we use the command `Solve[expr1 == expr2, unknown]`. For example, to solve for all x such that $x^2 + x - 1 = x^3$, we write:

```
Solve[x^2+x-1 == x^3, x]
```

To get numerical values, change `Solve` to `NSolve`. Find all the roots of the polynomial $x^3 - 2x^2 - x + 1$.

- (30) Recall that two vectors are perpendicular to each other if and only if their dot product is equal to zero. Find all possible values of a number c such that vector $ca + b$ is perpendicular to vector $a + cb$.
- (31) On paper, the magnitude (length) of vector a is written as $|a|$. In Mathematica, it is `Norm[a]`. Try the following:

```
Norm[a]
Norm[b]
```

then **Shift+Enter**. Check whether $|2a + 3b| = 2|a| + 3|b|$.

Recall that the angle $\theta \in [0, \pi]$ between two vectors a and b satisfies the equation

$$\cos \theta = \frac{a \cdot b}{|a||b|}$$

- (32) Knowing the cosine, one can find the angle using the function `ArcCos`. For example, if $\cos \theta = 0.45$ then θ is

```
ArcCos[0.45]
ArcCos[0.45]/Degree
```

The first command gives the value of the angle in radian. The second command gives the value in degree.

- (33) Find the measure in radian and degree of the angle between vectors $a = (4, 1)$ and $b = (3, 2)$.
- (34) To find the cross product between two vectors, we use the command `Cross`. Try the following:

```
a={1,2,3}
b={1,0,-1}
Cross[a,b]
```

- (35) Find the area of the triangle with vertices at $A(1, 2, 3)$, $B(2, 2, 1)$, and $C(0, -1, 3)$.