Lecture 10

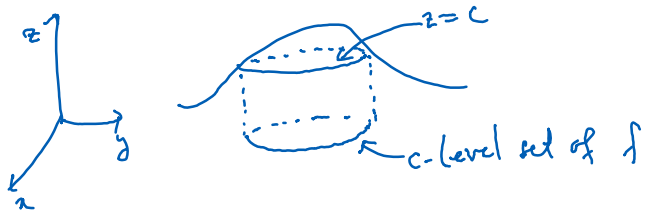
Wednesday, May 15, 2024 9:12 AM

* Graph of a function: $\{(x,y,z) \mid z = f(x,y)\}$ ← this is a surface in the space

Recall the graph of one-variable functions: $\{(x,y) \mid y = f(x)\}$ ← a curve on the plane

* Level sets of a function:

c-level set is $\{(x,y) \mid f(x,y) = c\}$



A contour map is a collection of many level sets.

* Limit of functions of more than one variable:

$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = a$ if $f(x,y)$ approaches a as (x,y) approaches (x_0,y_0) in any direction (within the domain).

Ex $f(x,y) = \frac{xy}{x^2+y^2}$

$\lim_{(x,y)\to(1,1)} f(x,y) = f(1,1) = \frac{1}{2}$

$\lim_{(x,y)\to(0,0)} f(x,y)$ DNE

Ex $f(x,y) = \dfrac{\sin(x^2+y^2)}{3x^2+2y^2}$, $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ DNE

Ex $f(x,y) = \dfrac{\sin(x^2+y^2)}{3x^2+3y^2}$, $\lim\limits_{(x,y)\to(0,0)} f(x,y) = \dfrac{1}{3}$

Ex $f(x,y) = \dfrac{xy}{x^2-y^2}$, $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ DNE

Note: L'Hospital rule may work for multivariable function, but it is subtle. Don't use it unless you understand its rule (see the supplement paper by Garg Lawlor of BYU)

Ex $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{x^2+y^2} = 0$ (squeeze thm)

$\lim\limits_{(x,y)\to(1,1)} \dfrac{e^x-e^y}{x-y} = ?$

Recall the mean value thm: $e^x - e^y = e^c(x-y)$ where c is between x and y.

$$\frac{e^x-e^y}{x-y} = e^c \longrightarrow e^1 = e \quad \text{as } (x,y)\to(1,1).$$

* Continuity of a function

* Partial derivatives ....