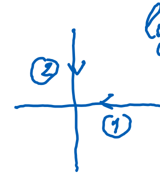
Lecture 11

Thursday, May 16, 2024 3:01 PM

$$f(x,y) = \frac{\sin(x^2+y^2)}{3x^2+2y^2}, \quad \lim_{(x,y)\to(0,0)} f(x,y) \quad \text{DNE}$$



$\lim_{①} = \frac{1}{3}$

$\lim_{②} = \frac{1}{2}$

Ex

$$f(x,y) = \frac{\sin(x^2+y^2)}{3x^2+3y^2}, \quad \lim_{(x,y)\to(0,0)} f(x,y) = \frac{1}{3}$$

Ex

$$f(x,y) = \frac{xy}{x^2-y^2}, \quad \lim_{(x,y)\to(0,0)} f(x,y) \quad \text{DNE}$$

Note: L'Hospital rule may work for multivariable function, but it is subtle. Don't use it unless you understand its rule (see the supplement paper by Gary Lawlor of BYU)

Ex

$$\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2+y^2} = 0 \quad \text{(squeeze thm)}$$

$$\lim_{(x,y)\to(1,1)} \frac{e^x - e^y}{x-y} = ?$$

Recall the mean value thm: $e^x - e^y = e^c(x-y)$ where c is between x and y.

$$\frac{e^x - e^y}{x-y} = e^c \longrightarrow e^1 = e \quad \text{as } (x,y) \longrightarrow (1,1).$$

* Continuity of a function