Lecture 12

Saturday, May 18, 2024 3:23 PM

Partial derivatives

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$g'(x,y) = ?$ (hard to define)

Instead, we will fix one variable and differentiate wrt the other variable:

$$\lim_{h \to 0} \frac{g(x+h,y) - g(x,y)}{h} = g_x(x,y),\ \frac{\partial g}{\partial x}(x,y),\ D_x g(x,y),\ D_1 g(x,y)$$

$$\lim_{h \to 0} \frac{g(x,y+h) - g(x,y)}{h} = g_y(x,y),\ \frac{\partial g}{\partial y}(x,y),\ D_y g(x,y),\ D_2 g(x,y)$$

These are called partial derivatives of g.

Ex $g(x,y) = xy^2 + e^{xy}$

Find g_x and g_y.

Ex $f(x,y,z) = \dfrac{xz}{x+yz}$

Find $f_x(1,1,2)$, $f_y(2,1,-3)$, $f_z(1,-1,2)$.

Higher derivatives

$(f_x)_x = f_{xx}$, $(f_x)_y = f_{xy}$, $(f_y)_x = f_{yx}$, $(f_y)_y = f_{yy}$

__Ex__ $f(x,y) = x^4 y^3$

Find $f_{xx}, f_{xy}, f_{yx}, f_{yy}$.

Notice that $f_{xy} = f_{yx}$.

$\boxed{\text{Clairaut's theorem: } f_{xy} = f_{yx}}$ (The order of differentiation doesn't matter.

Question: $f_{xyyx} \stackrel{?}{=} f_{yxxy}$

__Notations:__

$$f_{xx} = (f_x)_x = \left(\frac{\partial f}{\partial x}\right)_x = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2}$$

$$f_{xy} = (f_x)_y = \left(\frac{\partial f}{\partial x}\right)_y = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}$$

$$f_{xxzyyzxy} = \frac{\partial^8 f}{\partial x^3 \partial y^3 \partial z^2}$$