Lecture 13

Monday, May 20, 2024 2:33 PM

- Find $f_x(x,y)$: freeze y, diff. wrt x
- Find $f_x(1,2)$: two methods

  Method 1: find $f_x(x,y)$, then substitute $x=1$, $y=2$

  Method 2: plug $y=2$ into $f(x,y)$,
  then differentiate wrt x,
  then substitute $x=1$

Ex Let

$$f(x,y) = \frac{x}{x^2+y^2} e^{\frac{y}{x}}$$

Find $f_x(1,0)$ using both methods.

You will see that the second method is much simpler.

If you are asked to find $f_x(1,2)$, $f_x(2,3)$ then you probably should use the first method to avoid taking derivative multiple times.

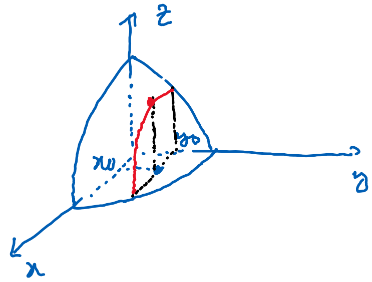
Note:

$f_{xx}(1,2)$: you can substitute $y=2$ into f, and then take derivative twice wrt x, and then substitute $x=1$.

$f_{xy}(1,2)$: you cannot substitude $x=1$ or $y=2$ before taking derivative.

Geometric meaning of partial derivative

$f'(x) =$ rate of change $=$ slope of tangent line.



$f_x(x_0, y_0) = g'(x_0)$ where $g(x) = f(x, y_0)$.

= rate of change of f at (x_0, y_0) along the line $y = y_0$

Ex $f(x,y) = 1 - x^2 - y^2$

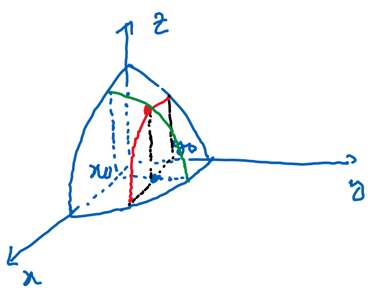
Explain why $f_x, f_y < 0$ for any $x, y > 0$, $x^2 + y^2 < 1$.

Compute f_x, f_y

* Tangent plane of a surface:

Along the red curve: $r_1(t) = (t, y_0, f(t, y_0))$

Tangent vector: $r_1'(x_0) = (1, 0, f_x(x_0, y_0))$



Along the green curve: $r_2(t) = (x_0, t, f(x_0, t))$

Tangent vector. $r_2'(y_0) = (1, 0, f_y(x_0, y_0))$

Take the cross product of those tangent vectors to get a normal vector of the tangent plane.