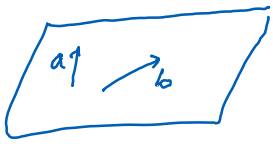
Lecture 14

Tuesday, May 21, 2024 12:21 PM

* Tangent plane (continued)

Two direction vectors: $a = (1, 0, f_x(x_0, y_0))$

$b = (0, 1, f_y(x_0, y_0))$



Normal vector: $n = a \times b$

$= (-f_x(x_0, y_0), -f_y(x_0, y_0), 1)$

Eq. of tangent plane:

$$-f_x(x_0, y_0)(x - x_0) + (-f_y(x_0, y_0))(y - y_0) + 1(z - z_0) = 0$$

Equivalently,

$$\boxed{z = z_0 + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)}$$

Ex find tangent plane to the graph of $f(x, y) = x^2 + 2y^2$ at $(2, 1)$.

Ex find tangent plane to the surface $xz + xz^2 + yz^3 = 1$ at the point $(-\frac{1}{2}, 2, 1)$.

Two methods: ① solve for y in terms of x and z

② Use implicit differentiation

Linear approximation

$$f(x, y) \approx \underbrace{f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)}_{L(x, y)}$$

$L(x, y)$: the linear function that best fits $f(x, y)$ near (x_0, y_0)

Ex $f(x,y) = x^3 + y^3$

Find linear approximation of f around $(1,2)$.

Ex Use linear approximation to approximate $\sqrt{(3.1)^2+(3.9)^2}$.

$f(x,y) = \sqrt{x^2+y^2}$

$f(3,4) = \sqrt{3^2+4^2} = 5$

$f(3.1, 3.9) \approx ?$