Lecture 15

Tuesday, May 21, 2024 2:38 PM

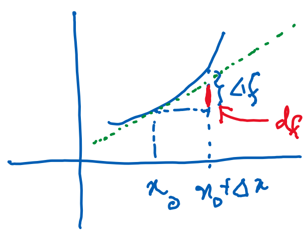
Linear approximation of $f(x)$ near x_0:

$$f(x) \approx \underbrace{\text{tangent line}}_{y = f(x_0) + f'(x_0)(x-x_0)}$$

Linear approximation of $f(x,y)$ near (x_0,y_0):

$$f(x,y) \approx \underbrace{\text{tangent plane}}_{z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)}$$

Differential



$\Delta f = f(x_0+\Delta x) - f(x_0) \approx$? (better than 0)

$\Delta f \approx df = f'(x_0)\Delta x$

Notation: $dx = \Delta x$

Thus, df = change in f after linear approximation.

df is called the differential of f.

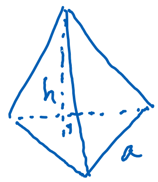
Now consider $f(x,y)$:

$$\Delta f = f(x_0+\Delta x, y_0+\Delta y) - f(x_0,y_0) \approx \underbrace{f_x(x_0,y_0)\Delta x + f_y(x_0,y_0)\Delta y}_{df}$$

Notation: $dx = \Delta x$, $dy = \Delta y$

Then $df = f_x(x_0,y_0)\,dx + f_y(x_0,y_0)\,dy$.

Ex



Tetrahedron with height h, the base being a regular triangle with side length a.

$$V = \frac{1}{3} h S = \frac{1}{3} h \left(\frac{\sqrt{3}}{4} a^2 \right) = \frac{\sqrt{3}}{12} h a^2$$

Assume the exact value of a is 5, the exact value of h is 8.

Assume a is measured with an allowable error of 0.1.

h " " " " " 0.2 (it is harder to measure h)

Find the maximum error in measuring the volume.

$$V(a,h) = \frac{\sqrt{3}}{12} h a^2$$

$$V(a,h) - V(5,8) = \Delta V \approx dV = V_a(5,8)\,da + V_h(5,8)\,dh$$

$$= \frac{\sqrt{3}}{12} 2(5)(8)\,da + \frac{\sqrt{3}}{12} 5^2\,dh$$

$$\leq \frac{\sqrt{3}}{12} 2(5)(8)\,0.1 + \frac{\sqrt{3}}{12} 5^2\,0.2$$

$$= \cdots$$