Lecture 16

Thursday, May 23, 2024 8:47 AM

Chain rule.

$$\frac{d}{dt} f(x,y) = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

Ex: given that $f_x(-2,0) = 2$ and $f_y(-2,0) = 3$. Find

$$\frac{d}{dt} f(2t, t^2-1) \quad \text{at } t = -1.$$

Here $x = 2t$ and $y = t^2 - 1$.

$$\frac{d}{dt} f(2t, t^2-1) = \frac{\partial f}{\partial x}(2t, t^2-1)\frac{d(2t)}{dt} + \frac{\partial f}{\partial y}(2t, t^2-1)\frac{d(t^2-1)}{dt}$$

$$= \frac{\partial f}{\partial x}(2t, t^2-1)\, 2 + \frac{\partial f}{\partial y}(2t, t^2-1)\,(2t)$$

Now substitute $t = -1$:

$$\left.\frac{d}{dt} f(2t, t^2-1)\right|_{t=-1} = \frac{\partial f}{\partial x}(-2,0)\, 2 + \frac{\partial f}{\partial y}(-2,0)(-2) = 2(2) + 3(-2) = -2.$$

Ex

$$x = 2t+1$$

$$y = t^2 - t$$

$$z = x^2 + xy + y^2 = z(t)$$

Find $z'(1)$ using two methods:

- Method 1 (without chain rule): substitute $x = 2t+1$, $y = t^2 - t$ into z and then differentiate wrt t, and then plug $t = 1$.
- Method 2: use the chain rule

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

More general chain rule:

$$\left.\begin{array}{l} z = f(x,y) \\ x = x(u,v) \\ y = y(u,v) \end{array}\right\} \quad \frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}, \qquad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}$$

Ex

$z = e^x \ln y$

$x = u + v$

$y = uv$

Find z_u and z_v.

Ex given $f_x(1,5,4) = 1$, $f_y(1,5,4) = 2$, $f_z(1,5,4) = 3$.

Find $\frac{\partial}{\partial u} f(uv, u+2v, 2u+v)$ at $(u,v) = (1,2)$.