Lecture 2

Thursday, May 2, 2024 10:35 PM

The set of all points with distance r from the point (a,b,c) is a sphere.

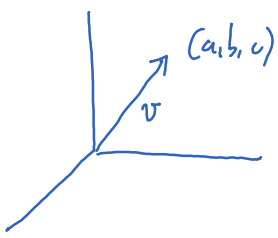
Equation of the sphere:

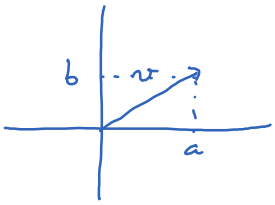
$$(x-a)^2+(y-b)^2+(z-c)^2=r^2$$

* Test if a point lies on / outside / inside a sphere
* Find the center and radius of the sphere with equation

$$x^2+y^2+z^2+2x-6y+8z-3=0$$

Vectors





A vector has an algebraic representation and a geometric representation.

$v=(a,b)$ or (a,b,c) — arrow (direction + length)

* Algebra on vectors

$v=(a,b)$

$+$ $w=(c,d)$

$v+w=(a+c,b+d)$

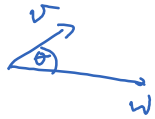
$2v=(2a,2b)$: scaling by 2

$-v=(-a,-b)$: scaling by -1

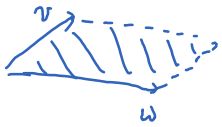
Geometrically, adding two vectors is equalent to the "triangle rule" or parallelogram rule.

Product: ~~$vw = (ac, bd)$~~ (not useful)

- Dot product – compute angle



- Cross product – compute area



* Dot product:

$$(a,b)\cdot(c,d) = ac+bd$$

$$(a,b,c)\cdot(d,e,f) = ad+be+cf$$

Properties:

- distributive: $u\cdot(v+w) = u\cdot v + u\cdot w$
- commutative: $u\cdot v = v\cdot u$