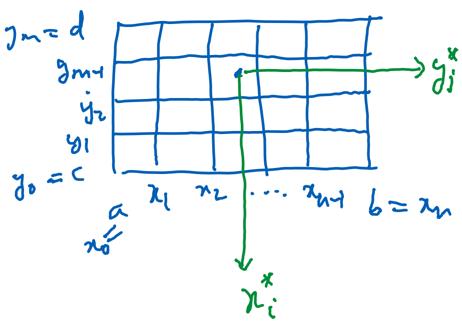
Lecture 21

Friday, May 31, 2024 1:49 PM



Volume of a tube $\approx$ volume of rectangular box

$= f(x_i^*, y_j^*)\,\Delta x\,\Delta y$

$\iint_D f(x,y)\,dA \approx$ sum of all $f(x_i^*, y_j^*)\,\Delta x\,\Delta y$.

Definition:

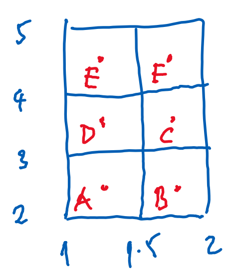
$$\iint_D f(x,y)\,dA = \lim_{m,n\to\infty} \sum_{i=1}^{n}\sum_{j=1}^{m} f(x_i^*, y_j^*)\,\Delta x\,\Delta y$$

if the limit exists.

Ex: Approximate

$$\iint_{[1,2]\times[2,5]} (x^2+y^2)\,dA$$

using $\Delta x = 0.5$, $\Delta y = 1$, and Midpoint rule.



$$\iint_{[1,2]\times[2,5]} (x^2+y^2)\,dA \approx f(A)\,0.5\,(1) + \cdots + f(F)\,0.5\,(1)$$

$$= \ldots \approx 45.68$$

There is a better way to compute. Notice that

$$\lim_{m,n\to\infty} \sum_i \sum_j f(x_i^*, y_j^*)\,\Delta x\,\Delta y = \lim_{m,n\to\infty} \sum_i \left(\sum_j f(x_i^*, y_j^*)\,\Delta y\right)\Delta x$$

$$= \lim_{m\to\infty} \sum_i \Bigl(\underbrace{\lim_{n\to\infty} \sum_j f(x_i^*, y_j^*)\,\Delta y}_{=\int_c^d f(x_i^*, y)\,dy}\Bigr)\Delta x = \int_a^b\left(\int_c^d f(x,y)\,dy\right)dx$$

* Fubini's theorem:

$$\iint_{[a,b]\times[c,d]} f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

Fubini's theorem is analogous to Clairaut's theorem: order of integration doesn't matter.

<u>Ex</u>

$$\iint_{[1,2]\times[2,5]} (x^2+y^2)\,dA = \int_1^2 \int_2^5 (x^2+y^2)\,dy\,dx = \int_1^2 \left(x^2 y + \frac{y^3}{3}\right)\Big|_{y=2}^{y=5} dx$$

$$= \int_1^2 (3x^2+39)\,dx = \left(x^3+39x\right)\Big|_1^2 = 46$$

<u>Ex</u>

$$\iint_{[1,2]\times[0,1]} \frac{x}{x+y}\,dA = \int_1^2 \int_0^1 \frac{x}{x+y}\,dy\,dx = \dots$$

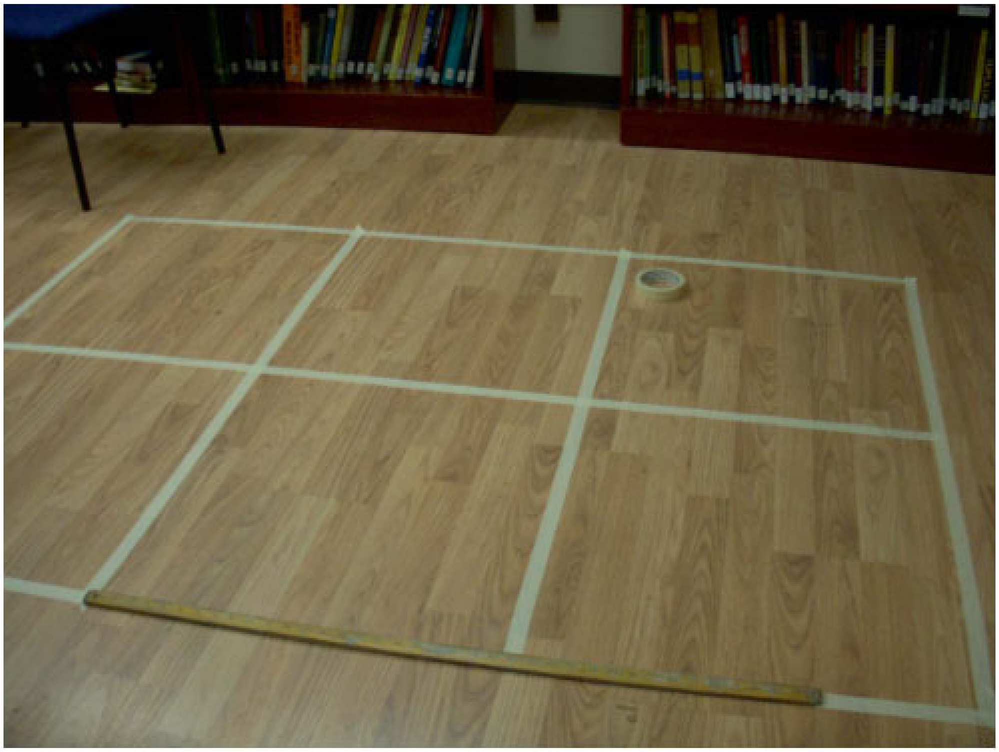


Figure 1. Masking tape used to create a grid. (Figure is provided in color online.)

Figure 2. Students standing in formation, creating surfaces. (Figure is provided in color online.)

From the article: "[Be the Volume: A Classroom Activity to Visualize Volume Estimation]" by Jessica Mikhaylov.